A New Distance Measure for Probabilistic Shape Modeling

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Abstract. The contour of a planar shape is essentially one-dimensional signal embedded in 2-D space; thus the orthogonal distance, which only considers 1-D (norm) deviation from suggested models, is not rich enough to characterize the description quality of arbitrary model/shape pairs. This paper suggests a generalized distance measure, called *Transport Distance*, for probabilistic shape modeling. B-Spline primitives are used to represent models. The probability of a hypothetical model for a shape is determined on the basis of the new distance measure. Experiments show that an optimization procedure, which maximize the model probability, generates robust and visually pleasing geometric models for data.

1 Introduction

The automatic estimation of geometric structures from shape data plays an important role in computer vision, since explicit analytic functions rather than point sets provide compact and analytically tractable representations of contours or surfaces. We assume in this paper that contour data are given as a chain code of a set of points in $\mathbb{R}^2$. Analytic representations of shapes can be flexibly incorporated in information systems for image segmentation, image compression, network transmission, object recognition, scientific visualization, virtual reality, content-based image/video retrieval, etc.[2].

Despite significant efforts, problems related to analytic shape representations remain still open. One key problem which has to be solved for geometric shape modeling is the question how to estimate the description quality of a hypothetical model for given data. Normally the *orthogonal distance*, which measures only deviations in normal direction of the geometric structure[1], is employed to estimated the description error. Similarity of shape, however, is not adequately captured by the orthogonal distance and we, therefore, replace this concept by an alternative deformation measure, the *transport distance*. In the new framework a data point on the contour is considered to be generated by moving a model point along a path either completely inside of the contour or completely outside of the contour. The restriction of the transport path to the interior or the exterior of the shape is supposed to model a topological constraint to ensure invariance to articulation of shapes like limb pose of animals and humans. The

B. Michaelis and G. Krell (Eds.): DAGM 2003, LNCS 2781, pp. 507–514, 2003.

most likely model is determined by the total transport of all model points to their corresponding contour points under the deformation constraint. This concept of shape modeling solves two conceptual problems which are related to the usage of orthogonal distance in probabilistic shape modeling: (i) construction of the model/data point correspondences; (ii) likelihood calculation from a point based distance measure.

As shown in Fig.1(b), it is difficult to decide if the individual data points on the tail part is a correct image of model points after orthogonal mapping. To avoid such poor correspondences, heuristic knowledge, such as high curvature point, inflexion points, stable scale etc. [3,5] are widely used to partition a shape into simple pieces, for which the orthogonal distance might be psychophysically motivated for piecewise model reconstruction. However, techniques using orthogonal distance together with heuristic shape partitioning exhibit the following two disadvantages: First, they almost always need some empirical thresholds or manual adjustments of parameters to compute the heuristic knowledge [5]. Second, they rarely provide a full optimization framework for shape model estimation [6], since it usually is difficult to formulate heuristic knowledge as model costs.

Many methods compute the likelihood based on the orthogonal distance from all available data points. Since the number of data points is sensitive to the noise, the likelihood is also noise dependent if all the data points are treated equally. Normalization only makes sense when the noise are homogeneously distributed along the contour. Weighing data points according to their local features, such as curvature, density, etc., faces difficulties in scale selection and weighing strategy. Alternative methods compute the likelihood from sampled model points [4]. But there are still problems in sampling ratio determination as well as distance measure for arbitrary model points. All these disadvantages are cured in the proposal to use the transport distance rather than the orthogonal distance for modeling shape deformation.

2 Related Works

For shape modeling techniques which use orthogonal distance together with heuristic knowledge based shape partitioning, readers might refer to the articles by Lindeberg and Li [3], and Bengtsson and Eklundh [5].

Splines are popularly adopted in geometric shape modeling [2]. Cham and Cippolla [4] suggested the *Potential for Energy-Reduction Maximization* (PERM) strategy to guide the new control points insertion in BSpline based shape modeling, but it is not transparently guided by the distance measure. Their paper discusses the data sampling problem in details.

Without the pre-process of data partitioning, Kern and Werman [7] have rebuilt the geometric structure of explicit functions of $y = f(x)$ or $z = f(x, y)$ using a fully Bayesian approach. Their work is distinguished from this paper since our targets are implicit 2-D functions.

3 Transport Probability

In this section, we will define the transport distance between two arbitrary $\mathbb{R}^2$ points and the transport probability between two curve segments.

Transport Path: Given two $\mathbb{R}^2$ points, o_s and o_t, one oriented and non-self-intersecting curve segment connecting them, is named a *transport path*, $\mathcal{A}(o_s, o_t; \boldsymbol{\alpha})$, from o_s to o_t, where $\boldsymbol{\alpha}$ specifies the curve segment in $\mathbb{R}^2$.

Transport Region: Given two oriented and non-self-intersecting $\mathbb{R}^2$ curve segments, $\mathcal{C}_s$ and $\mathcal{C}_t$, a region will be bounded by them if $\mathcal{C}_s$ and $\mathcal{C}_t$ connect the same point-pair and $\mathcal{C}_s$ does not intersect $\mathcal{C}_t$. (The intersecting cases will be discussed in next section.) This region together with its boundary, is called the *transport region*, $\mathcal{R}_{ts}$, for given $\mathcal{C}_s$ and $\mathcal{C}_t$. $\forall o_{cs} \in \mathcal{C}_s$ and $\forall o_{ct} \in \mathcal{C}_t$, $\exists \mathcal{A}(o_{cs}, o_{ct}; \boldsymbol{\alpha}) \subset \mathcal{R}_{ts}$. Assuming that $\mathcal{C}_s$ has curve length l_s, $o_{cs} \in \mathcal{C}_s$ can be parameterized by its arclength position u_s, where $0 \leq u_s \leq l_s$.

Path Bundle and Transport Front: Our target is to measure the deformation distance of $\mathcal{C}_t$ from $\mathcal{C}_s$. Instead of constructing the one-to-one mapping between these two curve segments, we map the transport region, $\mathcal{R}_{ts}$, onto $\mathcal{C}_s$ while assuming that $\mathcal{R}_{ts}$ can be generated by transporting individual points $o_{cs} \in \mathcal{C}_s$. Once $\mathcal{R}_{ts}$ is generated, $\mathcal{C}_t$ will be obtained. It is assumed that there is a set of maps $\{h\} = \mathcal{H}_{ts}$, in which $h \in \mathcal{H}_{ts}$ maps individual points $o_r \in \mathcal{R}_{ts}$ to (u_h, v_h), where u_h denotes the index of individual paths, and v_h denotes the positions on a path. To ensure that the mapping is unique and continuous, the following conditions are proposed to constrain $\mathcal{H}_{ts}$:

1. **one-to-one-mapping-condition**: $\forall o_r \in \mathcal{R}_{ts}$, there exists a mapping $h : o_r \mapsto (u_h, v_h)$ $u_h \geq 0$ and $v_h \geq 0$. $o'_r \neq o_r$ iff $(u'_h, v'_h) \neq (u_h, v_h)$;
2. **source-condition**: all $o_{cs} \in \mathcal{C}_s$ will be mapped to $(u_h = u_s, v_h = 0)$;
3. **no-breaking-path-condition**: $\forall o_r = (u_h, v_h) \in \mathcal{R}_{ts}$, $\forall \epsilon > 0$, $\exists o'_r \in \mathcal{R}_{ts}$: $o'_r = (u_h, v'_h)$ holds $0 \leq v_h - v'_h < \epsilon$. This condition ensures that all $o_r \in \mathcal{R}_{ts}$ are transported from $\mathcal{C}_s$;
4. **continuity-condition**: $\forall o_r \in \mathcal{R}_{ts}$, $\forall \epsilon > 0$, $\exists \delta > 0$ determining an open ball $\mathcal{N}_e(o_r, \delta) \subset \mathbb{R}^2$, such that $o'_r : o'_r \in \mathcal{N}_e \cap \mathcal{R}_{ts}$ holds $\sqrt{(u_h - u'_h)^2 + (v_h - v'_h)^2} < \epsilon$. This condition ensures that within $\mathcal{R}_{ts}$, u_h and v_h are differentiable;
5. **target-condition**: $\forall o_r = (u_h, v_h) \in \mathcal{R}_{ts}$, $\exists o_{ct} \in \mathcal{C}_t : o_{ct} = (u_h, v_h^{(t)})$ satisfying $v_h \leq v_h^{(t)}$, so that every path leads to $\mathcal{C}_t$;
6. **locally-Euclidean-condition**: $\forall o_r \in \mathcal{R}_{ts}$, $\exists \delta > 0$ determining an open neighborhood $\mathcal{N}_e(o_r, \delta) \subset \mathbb{R}^2$, such that all $o'_r \in \mathcal{N}_e \cap \mathcal{R}_{ts}$ can be parameterized by a locally *Cartesian* coordinate system as $o'_r = (u'_e, v'_e)$ (and $o_r = (u_e, v_e)$). The *Euclidean* distance of o'_r from o_r is then measured as $d_e(o'_r, o_r) = \sqrt{(u'_e - u_e)^2 + (v'_e - v_e)^2}$. Moreover, there should exist two scalar factors, $w_u = \lim_{\delta \to 0}(u'_h - u_h)/(u'_e - u_e) = du_h/du_e$ and $w_v = \lim_{\delta \to 0}(v'_h - v_h)/(v'_e - v_e) = dv_h/dv_e$. So that within $\mathcal{N}_e$, the *Euclidean* distance measure can be approximated by u_h and v_h.

Given $h \in \mathcal{H}_{ts}$, $\mathcal{R}_{ts}$ is decomposed as a set of transport paths

$$h : \mathcal{R}_{ts} \mapsto \mathcal{D}_{ts}^{(h)} = \{\mathcal{A}_r^{(h)}(u_h) | 0 \leq u_h \leq l_s\} \quad (1)$$

where $\mathcal{A}_r^{(h)}(u_h) = \{o_r | h : o_r \mapsto (u_h, v_h)\}$ is a transport path which originates from $o_{cs}(u_s) = (u_h, 0)$ on $\mathcal{C}_s$, and ends at $o_{ct}(u_t) = (u_h, v_h^{(t)})$ on $\mathcal{C}_t$. $\forall u_h \neq u'_h$ holds $\mathcal{A}_r^{(h)}(u_h) \cap \mathcal{A}_r^{(h)}(u'_h) = \emptyset$. The set $\mathcal{D}_{ts}^{(h)}$ is called a *path bundle* of $\mathcal{R}_{ts}$. Each value of v_h determines a *transport front*, $\mathcal{F}_r^{(h)}(v_h) = \{o_r | h : o_r \mapsto (u_h, v_h)\}$. $\forall v'_h \neq v_h$ holds $\mathcal{F}_r^{(h)}(v_h) \cap \mathcal{F}_r^{(h)}(v'_h) = \emptyset$.

Transport Distance: Given two $\mathbb{R}^2$ points, a *transport distance* between them, $V(o_s, o_t; \boldsymbol{\alpha})$, is defined as the curve length of a particular transport path $\mathcal{A}(o_s, o_t; \boldsymbol{\alpha})$. We have $V(o_s, o_t; \boldsymbol{\alpha}) = \int_{\mathcal{A}(o_s, o_t; \boldsymbol{\alpha})} dv$ where v denotes the arclength measure.

Probability Density from Stepwise Transport: It is assumed that the probability density of a transport path, $p\left(\mathcal{A}(o_s, o_t; \boldsymbol{\alpha})\right)$, only depends on its transport distance according to a continuous distribution $p(v_d; \boldsymbol{\theta})$, where v_d is a random variable and $\boldsymbol{\theta}$ denotes a parameter vector. Defining $p'_{log}(v_d; \boldsymbol{\theta}) = d\left(\log p(v_d; \boldsymbol{\theta})\right)/dv_d$, the negative logarithm of $p\left(\mathcal{A}(o_s, o_t; \boldsymbol{\alpha})\right)$ can be calculated by

$$\begin{aligned} -\log p\left(\mathcal{A}(o_s, o_t; \boldsymbol{\alpha})\right) &= -\int_{\mathcal{A}(o_s, o_t; \boldsymbol{\alpha})} p'_{log}(v; \boldsymbol{\theta}) dv - \log p(0, \boldsymbol{\theta}) \\ &= -\lim_{\Delta_a \to 0} \sum_{o_v \in \mathcal{A}(o_s, o_t; \boldsymbol{\alpha})} p'_{log}(v(o_a); \boldsymbol{\theta}) \Delta_a - \log p(0, \boldsymbol{\theta}) \quad (2) \end{aligned}$$

where v is the arclength, $\{o_v\}$ are sampled points and Δ_a is the sampling rate.

Locally Distinguishable Paths: Given a transport step Δ_v, the path bundle can be obtained by incrementally prolongating individual paths from $\mathcal{C}_s$ to nearby points in $\mathcal{R}_{ts}$. Given a resolution $\varepsilon_{re} > 0$, two transport paths, $\mathcal{A}_r^{(h)}(u_h)$ and $\mathcal{A}_r^{(h)}(u'_h)$ are locally distinguishable on $\mathcal{F}_r^{(h)}(v_h + \Delta_v)$, if they satisfy one of the following two conditions: 1), they are distinguishable on the upper front $\mathcal{F}_r^{(h)}(v_h)$; 2) the *Euclidean* distance between $o_r = (u_h, v_h + \Delta_v)$ and $o'_r = (u'_h, v_h + \Delta_v)$, holds $d_e(o_r, o'_r) \geq \varepsilon_{re}$. Defining Δ_n as

$$\Delta_n(o_r) = \min\left(\Delta_n(o_r^{up} = (u_h, v_h)), w_u(o_r) \times \varepsilon_{re}\right), \quad (3)$$

where $o_r^{up} = (u_r, v_r)$, the closest two neighbors of $\mathcal{A}_r^{(h)}(u_h)$ on $\mathcal{F}_r^{(h)}(v_h + \Delta_v)$ will be $\mathcal{A}_r^{(h)}(u_h \pm \Delta_n(o_r))$. Given a small 1-D neighborhood, $\Delta_u \geq \Delta_n(o_r)$ around u_h, the number of locally distinguishable neighbors of $\mathcal{A}_r^{(h)}(u_h)$ will be $N_n(o_r) \approx \Delta_u / \Delta_n(o_r)$. Taking Δ_u as the sampling rate, we sample $I^{(v_h + \Delta_v)}$ points from $\mathcal{F}_r^{(h)}(v_h + \Delta_v)$ giving $\{o_r^{(i)} | i = 1, 2, ..., I^{(v_h + \Delta_v)}\}$. The total number of distinguishable stepwise transports, which are necessary to prolongate the path bundle from $\mathcal{F}_r^{(h)}(v_h)$ to $\mathcal{F}_r^{(h)}(v_h+\Delta_v)$, will be counted by

$$N_s(\mathcal{F}_r^{(h)}(v_h),\Delta_v)) \approx \sum_{i=1}^{I^{(v_h+\Delta_v)}} N_n(o_r^{(i)}) = \sum_{i=1}^{I^{(v_h+\Delta_v)}} \frac{1}{\Delta_n(o_r^{(i)})}\Delta_u. \tag{4}$$

For a fixed range of u_h, we have $N_s(\mathcal{F}_r^{(h)}(v_h+\Delta_v),\Delta_v)) \geq N_s(\mathcal{F}_r^{(h)}(v_h),\Delta_v))$.

Transport Probability for Path Bundle: Assuming that transport paths in a path bundle will be distinguished only locally, we define the stepwise probability from $\mathcal{F}_r^{(h)}(v_h)$ to $\mathcal{F}_r^{(h)}(v_h+\Delta_v)$ as

$$P_{log}^{(s)}\left(\mathcal{F}_r^{(h)}(v_h),\mathcal{F}_r^{(h)}(v_h+\Delta_v)\right) \approx p'_{log}(v_h;\boldsymbol{\theta})\Delta_v N_s(\mathcal{F}_r^{(h)}(v_h),\Delta_v)). \tag{5}$$

Extended from Eq.2, the negative logarithm of the probability of a path bundle (which is generated from J_f stepwise prolongations) is calculated as

$$\begin{aligned}
&-\log P\left(\mathcal{C}_s,\mathcal{C}_t;\mathcal{D}_{ts}^{(h)}\right) \\
&= -\sum_{j=0}^{J_f-1} P_{log}^{(s)}\left(\mathcal{F}_r^{(h)}(j\Delta_v),\mathcal{F}_r^{(h)}((j+1)\Delta_v)\right) - I^{(0)}\log p(0,\boldsymbol{\theta})\Delta_u \\
&\approx -\sum_{j=0}^{J_f-1}\left(\sum_{i=1}^{I^{((j+1)\Delta_v)}}\frac{1}{\Delta_n(o_r^{(i)})}\Delta_u p'_{log}(j\Delta_v;\boldsymbol{\theta})\Delta_v\right) - I^{(0)}\log p(0,\boldsymbol{\theta})\Delta_u \\
&\approx -\sum_{j=0}^{J_f-1}\sum_{i=1}^{I^{((j+1)\Delta_v)}}\frac{1}{\Delta_n(o_r^{(i)})}p'_{log}(j\Delta_v;\boldsymbol{\theta})\Delta_u\Delta_v - I^{(0)}\log p(0,\boldsymbol{\theta})\Delta_u
\end{aligned} \tag{6}$$

According to the *locally-Euclidean-condition*, we sample points in $\mathcal{R}_{ts}$ by a sampling rate Δ_e, and calculate the negative logarithm of the probability as

$$\begin{aligned}
&-\log P\left(\mathcal{C}_s,\mathcal{C}_t;\mathcal{D}_{ts}^{(h)}\right) \\
&\approx -\sum_{o_r\in\mathcal{R}_{ts}}\frac{w_u(o_r)w_v(o_r)}{\Delta_n(o_r)}p'_{log}(v_h(o_r);\boldsymbol{\theta})\Delta_e\Delta_e - \sum_{o_r\in\mathcal{C}_s}\log p(0,\boldsymbol{\theta})w_u(o_r)\Delta_e.
\end{aligned} \tag{7}$$

4 Probabilistic Shape Modeling

Given a closed and non-self-intersecting planar curve model $\mathcal{M}$, and a closed boundary $\mathcal{B}$, a number of intersecting points[1], $\{o_k|k=1,2,...,K\} \subseteq \mathcal{M}\cap\mathcal{B}$,

[1] If there is no intersecting point, then global geometric transformations, i.e., scaling, rotation, or translation are needed to approximately align the model and the given boundary.

can always be found to naturally partition both the data and the model into K pieces

$$\mathcal{M} = \{M_k(o_k, o_{k+1}) | k = 1, 2, ..., K\}, \tag{8}$$

$$\mathcal{B} = \{B_k(o_k, o_{k+1}) | k = 1, 2, ..., K\} \tag{9}$$

where $o_{K+1} = o_1$ and the coincide piece-pair (M_k, B_k) satisfies that $M_k \setminus \{o_k, o_{k+1}\} \cap B_k \setminus \{o_k, o_{k+1}\} = \emptyset$. Thus M_k and B_k bound their *transport region* $R_k = \mathcal{R}_{ts}(M_k, C_k)$. Each region will be either fully inside the model shape, or fully outside the model shape. Given a mapping method, we will have a path bundle $h : R_k = \mathcal{R}_{ts}(M_k, B_k) \mapsto \mathcal{D}_{ts}^{(h)}(R_k)$.

As shown in Fig.1(a), there might be multiple permitted correspondences using different intersecting points. Suppose that for a particular model/shape pair, there is a set of possible correspondences $\{\gamma\} = \Gamma$. For a particular case γ, we have a region set $\{R_{\gamma,k} | k = 1, 2, ...K_\gamma\}$ with their path bundles $\{\mathcal{D}_{ts}^{(h)}(R_{\gamma,k})\}$. According to Eq.7, the description quality is then estimated from individual transport regions

$$-\log P\left(\mathcal{B}|\mathcal{M}\right) = \min_{\gamma \in \Gamma, h \in \mathcal{H}_{ts}} \sum_{k=1}^{K_\gamma} -\log P\left(M_{\gamma,k}, B_{\gamma,k}; \mathcal{D}_{ts}^{(h)}(R_{\gamma,k})\right) \tag{10}$$

5 Experimental Results

We assume that the statistical properties of transport distance between a hypothetic model and the given data are characterized by a *Gaussian*. Three experiments are designed to demonstrate our ideas. Firstly, we generate an artificial shape and try to re-model it according to a dynamic process which minimize the negative logarithms of the model/contour probability. As shown in Fig.2(a) and (b), we precisely re-built the model starting from a poor initialization. Secondly, we model a real camel shape by B-Spline primitives (Fig.2(c))(also starting from the poor circle-like initialization). Thirdly, we model a complex see-horse shape using B-Spline primitives. It is clearly demonstrated that the description quality degrades when fewer control points are used (Fig.3).

In our experiments, a dynamical process is used to modify the position of individual control points of B-Splines. All the possible modifications of a model are suggested by the transport distance based probability, which is after the construction of a path bundle for individual transport regions between the hypothetical model and the given data. Currently we rank all the transport probability of individual transport regions between model and data, and assign update priorities to corresponding model pieces. For each visual piece, 4 related B-Spline primitives can be modified since we use B-Splines of order 3.

6 Discussion and Conclusion

In this paper we focus on the problem of a distance measure and a probability definition of hypothetical models for given data. Depending on the geometric

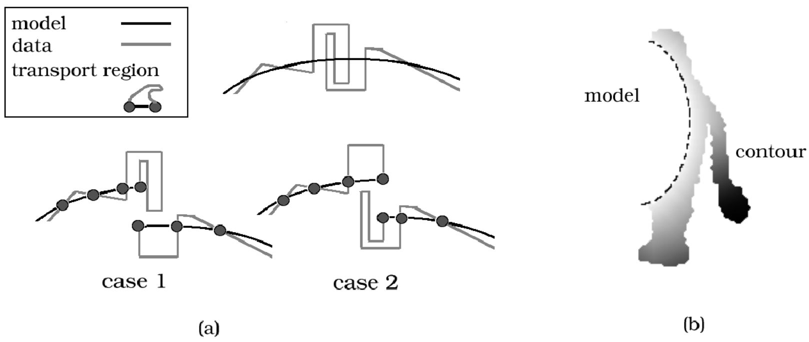

Fig. 1. transport region and transport distance: (a), complicated one to one piecewise model/data correspondences: both cases are permitted; (b), transport distance: from the model to the contour, the darker color, the longer transport distance.

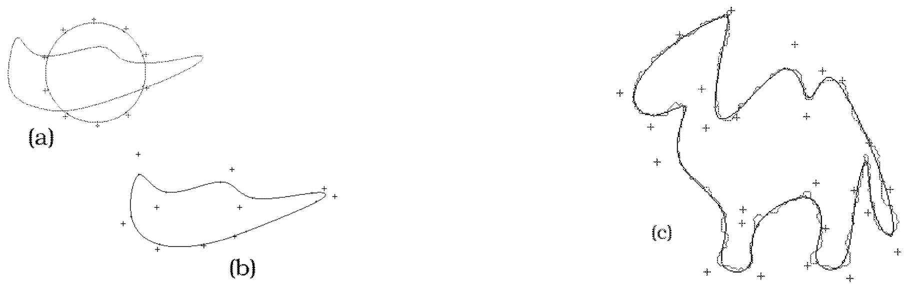

Fig. 2. Experimental results(1, 2): (a), artificial data generated from b-spline model and the poor initialization; (b): precisely recovered geometric model; (c), model reconstruction of a camel shape.("+" denotes the control points of a BSpline model.)

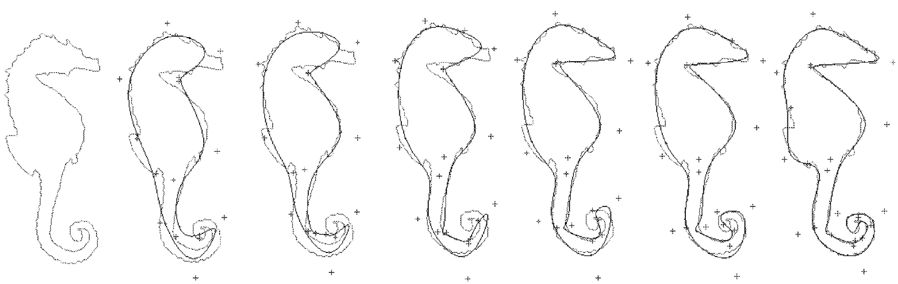

Fig. 3. Experimental results 3: model quality vs. model complexity. On the left, the source contour is shown.

properties of the input contour, the differences between the transport distance and the orthogonal distance are more or less pronounced accoring to the topological properties of individual transport regions between model and data.

Naturally we avoid the data pre-processing problem which is always required by orthogonal distance based methods when given shapes are complex. We are not arguing that heuristic knowledge is inappropriate for shape modeling, but we prefer to avoid it in the modeling process and we rather use shape related knowledge, i.e, curvature derivatives, arclength measurements, scaling, for tasks such as recognition, matching, etc. These measures are conveniently defined on the basis of analytic functions.

Although we only use a *Gaussian* assumption in experiments, other distributions can also be considered to characterize the statistical properties of transport distance. For example, if $p(v_d, \boldsymbol{\theta})$ is a *Laplace* distribution, then the area of a transport region provides the lower bound of Eq.7.

The robustness of model reconstruction originate from the fact that for a hypothetical model, its transport path bundle are determined by the topological properties of individual transport regions, which is robust to unknown small noise along the given data.

In this paper we suggest a new generalized distance measure, the transport distance, for geometric shape reconstruction. Experiments demonstrate that the transport distance can effectively characterize the model quality and guide the model selection.

The approach presented provides a suitable starting point for several further extensions: first, the tradeoff between model complexity and model quality should be thoroughly investigated; second, transport distance based shape modeling should be combined with other vision tasks such as segmentation, recognition, etc.; third, the concept of transport distance should be extended to higher dimensions such as surface modeling.

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