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Deadline: Beginning of next lecture

Algorithmic Game Theory

Autumn 2021

Exercise Set 10

These exercises are **non-graded**. Please submit your your solutions via moodle, in order to get **feedbacks**, before (**Dec 6, 10:00 am** – longer deadline). If you cannot use moodle, please send solutions by **email** to agt-course@lists.inf.ethz.ch.

Exercise 1: (2 Points)
Construct a game G which is NBR-solvable and such that asynchronous best-response take $\Omega(n)$ rounds to converge to a pure Nash equilibrium, where n is the number of players.

Exercise 2: (2+2 Points)
Consider the following **simpler** definition of NBR-solvable game with clear outcome, where we exchanged the "quantifier" between "elimination sequence" and "player i" (compare it with the original definition in lecture notes 10):

Definition 1 (NBR-solvable with clear outcome (simpler)) *A NBR-solvable game G has a clear outcome if there exists a tie breaking rule $\prec$ such that the following holds. There exists an elimination sequence consisting of players $p_1, \ldots, p_a, \ldots, p_\ell$ and strategies $E_1, \ldots, E_a, \ldots, E_\ell$ (according to Definition 4 in lecture notes) such that, for every player i the following holds:*

1. *p_a denotes the* first appearance *of i in the sequence, that is,*

$$p_a = i \neq p_1, p_2, \ldots, p_{a-1};$$

2. *in the corresponding subgame*

$$G_{a-1} = G \setminus (E_1 \cup E_2 \cup \cdots \cup E_{a-1})$$

the PNE s^* is globally optimal for *i, that is,*

$$u_i(\hat{s}) \leq u_i(s^*) \quad \textit{for all } \hat{s} \in G_{a-1}.$$

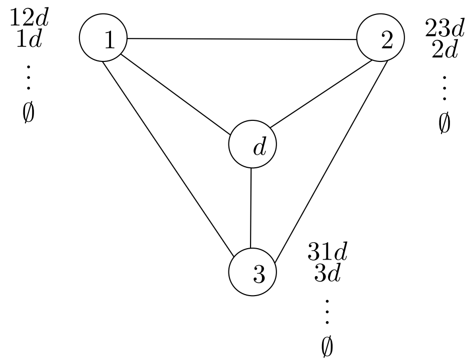
(Recall that s^* is the unique profile in the final subgame G_ℓ.)

Your task is to exhibit a game such that

- Asynchronous best-response converge and are incentive compatible;
- The game does **not** satisfy the definition above.

Exercise 3: (3 Points)

Recall from the lecture notes the **Gao-Rexford model** for the preferences in BGP games. Consider the following network and preferences:



Suppose that **1 is a customer of 2** and that all other pairwise commercial relationships (not specified) are such that, with the preferences above, do not violate **GR1** (customer-paths over peer-paths over provider-paths) and **GR2** (transit traffic). Show that the third condition **GR3 is violated**.