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Deadline: Beginning of next lecture

Algorithmic Game Theory

Autumn 2021

Exercise Set 4

These exercises are **non-graded**. Please submit your your solutions via moodle, in order to get **feedbacks**, before the beginning of next lecture (**October 22, 10:00 am**). If you cannot use moodle, please send solutions by **email** to agt-course@lists.inf.ethz.ch.

Exercise 1: (2 + 2 + 2 Points)

Consider the following cost-minimization game, played between two drivers approaching a crossing. The drivers can either Cross or Stop. If they Stop there is some small cost but if they both Cross then there is a much higher cost (crash).

	Cross	Stop
Cross	100 100	1 0
Stop	0 1	1 1

Traffic Light Game

The numbers in the game above are **costs** for the players.

(a) List all pure and mixed Nash equilibria.

(b) Give a correlated equilibrium that is not a mixed Nash equilibrium.

(c) Argue that in this game the set of coarse correlated equilibria coincides with the set of correlated equilibria.

Exercise 2: (2 Points)

The multiplicative-weights algorithm presented in the lecture was stated such that the overall length of the sequence T is given as a fixed parameter. Give a no-external regret algorithm that works without such a parameter for all possible T.

Hint: Use the algorithm from class as a subroutine (you do not need to analyze it again). Start with $T = 1$ as a guess and run the subroutine. Once the subroutine ends, restart it but double your guess.

Exercise 3: (4 Points)

Consider the following algorithm for minimizing external regret (the possible actions are $\{1, 2, \ldots, m\}$ and the costs of each action at time t is always 0 or 1):

Greedy Algorithm (GR)

- Initially set $p^1(a) = 1$ for $a = 1$ and $p^1(a) = 0$ otherwise.
- For each time step $t = 1, \ldots, T$:
  - Let $C^t_{BEST} = \min_a C^t(a)$ and let $S^t = \{a \mid C^t(a) = C^t_{BEST}\}$, where
$$C^t(a) = c^1(a) + c^2(a) + \cdots + c^t(a) \ .$$
  - Set $p^{t+1}(a) = 1$ for $a = \min S^t$ and $p^{t+1}(a) = 0$ otherwise.
    (Choose the minimum element in subset S^t of actions.)

Show that the expected cost achieved by this algorithm on any sequence of T steps is bounded as follows:
$$C^T_{GR} \leq m \cdot C^T_{BEST} + (m - 1)$$
where m is the number of possible actions, and $C^T_{GR} = \sum_{t=1}^{T} \sum_a p^t(a)c^t(a)$ is the expected of the algorithm.