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Deadline: Beginning of next lecture

# Algorithmic Game Theory

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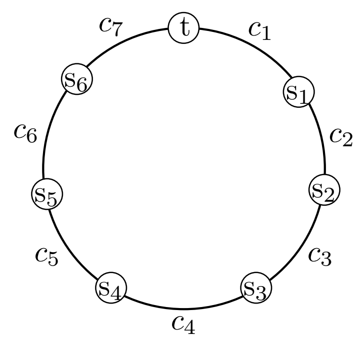
Exercise Set 5

**Exercise 1:** (3 Points)

Consider the following fair cost-sharing game:




Each player $i$ wants to be connected from $\mathsf{s}_i$ to $\mathsf{t}$, and the edges are **undirected**. That is, two or more players can use the same edge $e$ in different directions, and they all share equally the cost $c_e$ of this edge.

Prove that the price of stability for pure Nash equilibria is at most 2 in this game. (In fact, you can consider any ring with $n$ players.)

**Exercise 2:** (2 Points)

Consider the following mechanism $(A, P)$ for the shortest path problem (see lecture notes for the setting):

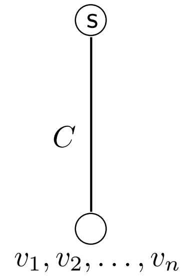
(a) Algorithm $A$ computes the shortest path according to the reported costs $c = (c_1, \ldots, c_n)$.

(b) Each selected player $i$ (i.e., a player whose edge is in the selected path) receives a payment $P_i$ equal to the cost of the alternative shortest path, that is, the length of the shortest path other than the path $A(c)$ computed in the previous step.

Show that this mechanism is **not truthful** on general graphs.

**Exercise 3:** (3 Points)

Consider the following problem: There are $n$ users (players) potentially interested in receiving a TV transmission, and the transmission is sent from a server $\mathsf{s}$ over this simple network:




All users are located in the same node, and $C > 0$ is the **cost** for sending the transmission to one or more users:

> *The server* s *can select which users receive the transmission (and which do not). If one or more receive,* s *has cost exactly $C$ in total (the use of the link). If no user receives, the cost for* s *is* 0*.*

Each user has a **private valuation** $v_i$, that is, how much he/she is willing to pay for the transmission. Users can chet and report a different valuation $r_i$. The **utility** of user $i$ is given by the difference between the valuation and the amount of money he/she has to pay:

$$-P_i + \begin{cases} v_i & \text{if } i \text{ receives the transmission} \\ 0 & \text{otherwise} \end{cases}$$

Give a truthful mechanism satisfying the following two conditions:

1. Either only the user with highest (reported) valuation receives and none else does, or none receives the transmission.

2. If one user receives, then he/she is charged an amount which is at least the cost $C$.

Truthful means that reporting the true valuation $v_i$ maximizes $i$'s utility (no matter how we fix the reports $r_j$ of the other users).