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Deadline: Beginning of next lecture

Algorithmic Game Theory

Autumn 2021

Exercise Set 8

These exercises are **non-graded**. Please submit your your solutions via moodle, in order to get **feedbacks**, before the beginning of next lecture (**Nov 19, 10:00 am**). If you cannot use moodle, please send solutions by **email** to agt-course@lists.inf.ethz.ch.

Exercise 1: (1 Points)

Consider running the **Serial-Dictator** mechanism for the Stable-Matching problem. Show that, in some cases, the output of Serial-Dictator is **not in the core**.

Exercise 2: (4 Points)

Consider the following facility location problem. We have N feasible locations on the line correspoding to the points $\{1, 2, \dots, N\}$. There are n players having an ideal (private) position p_i where they would like the facility to be opened, and their cost if facility x is chosen is the distance to the facility $c_i(x) = |x - p_i|$.

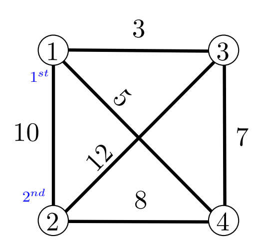
Question: Give an incentive compatible (truthful) mechanism which guarantees a **2-approximation** for the **maximum** cost

$$maxcost(x, p) = \max_i c_i(x),$$

where $p = (p_1, \dots, p_n)$ and $c_i()$ is as above. (The solution should have *maxcost* at most twice the optimal one, no agent should benefit from misreporting p_i, and there are no payments.)

Exercise 3: (4 Points)

In **correlated markets** we have a complete weighted graph. Each player (node) prefers neighbors whose edges weights are higher, as shown in this small example:



That is, in a correlated market players have a restricted set of preferences over the other nodes (we still want to match nodes to form a matching). These instances satisfy the following interesting property:

Acyclic Instances: There is no cycle of $\ell \geq 3$ players

$$i_1 \to i_2 \to \cdots \to i_\ell \to i_1$$

such that each player prefers the next one over the previous one.

Prove that indeed any instance of correlated markets (any complete graph) is acyclic.

Exercise 4: (1 Points)
If we run the TTCA on correlated markets, suitably adapted, where at each step we consider the graph of "top preferences" like for the House Allocation problem (each node points to its top choice among the "remaining ones"). Does the algorithm find a stable matching? (motivate your answer).

Note: Since the graph is not bipartite, we want that no two nodes that are unmatched in the computed solution, can both strictly improve by being matched to each other.