Optimal dislocation with persistent errors in subquadratic time

Barbara Geissmann, Stefano Leucci,
Chih-Hung Liu, Paolo Penna

ETH Zurich

Computing with Errors

Hardware faults

impossible to avoid

Computing with Errors

Hardware faults

impossible to avoid

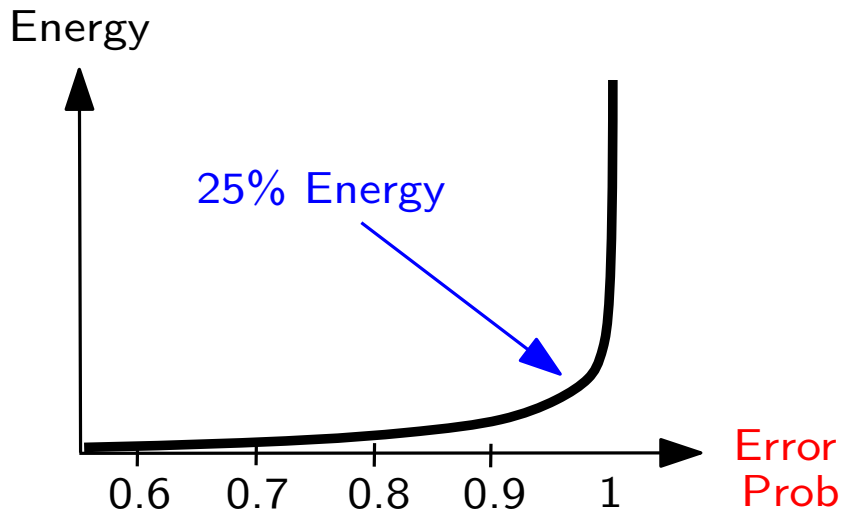
still want correct result

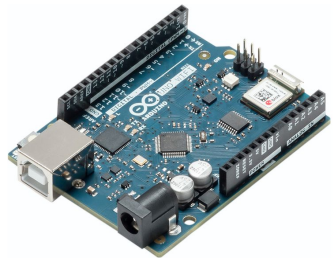
Computing with Errors

Hardware faults

impossible to avoid

introduced deliberately





Probalilistic CMOS

Computing with Errors

Hardware faults

impossible to avoid

introduced deliberately

Some human component

web search,
crowdsourcing,
social choice,
peer grading,...

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Sorting with erroneous comparisons

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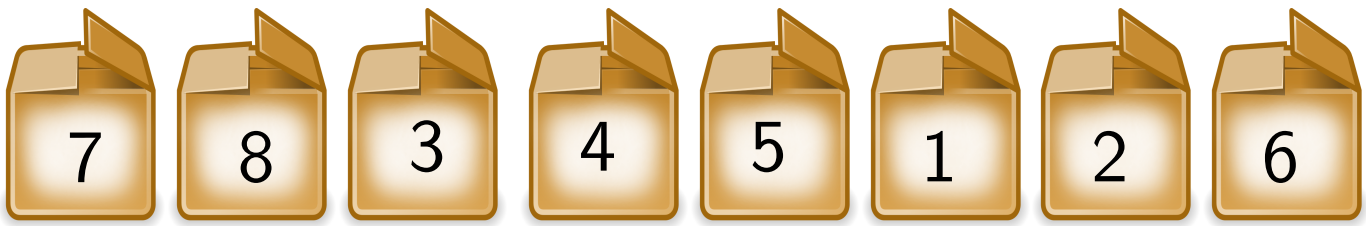

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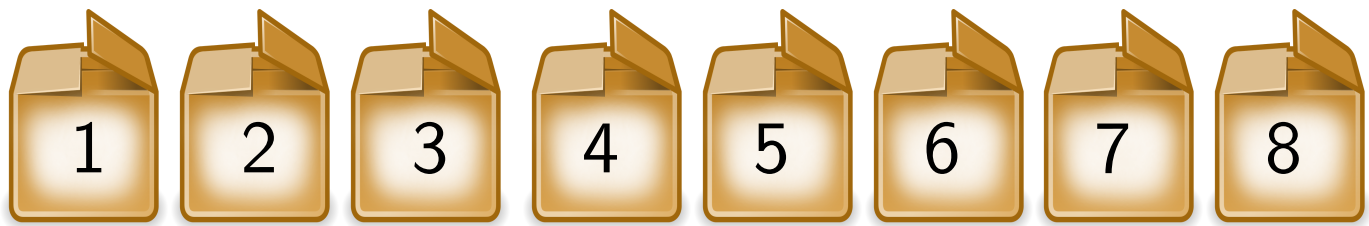
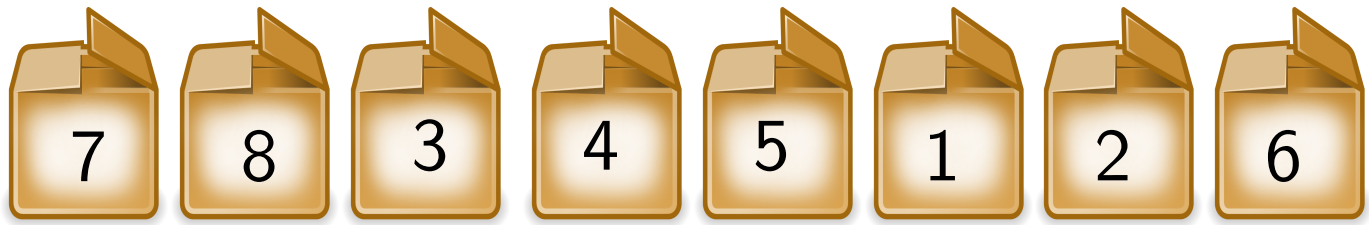


The Problem



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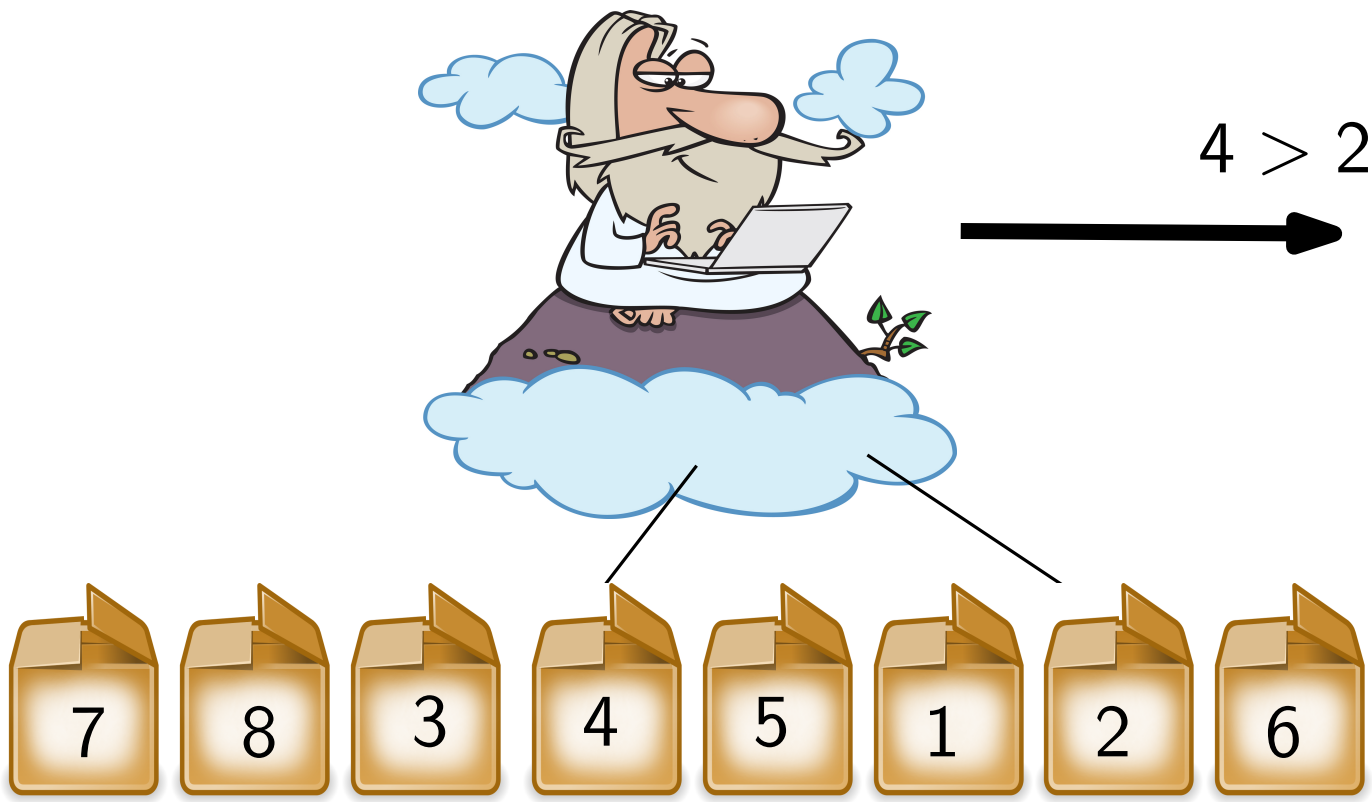

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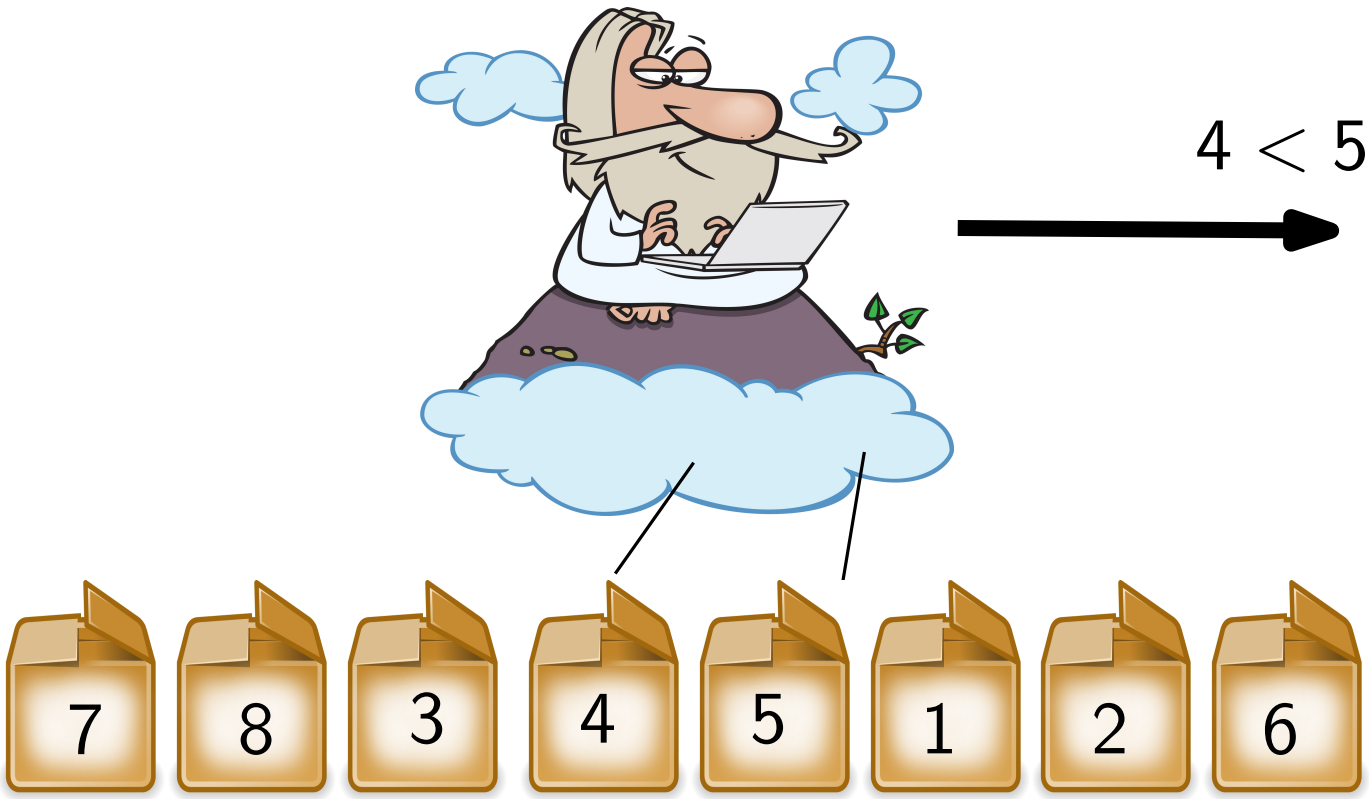

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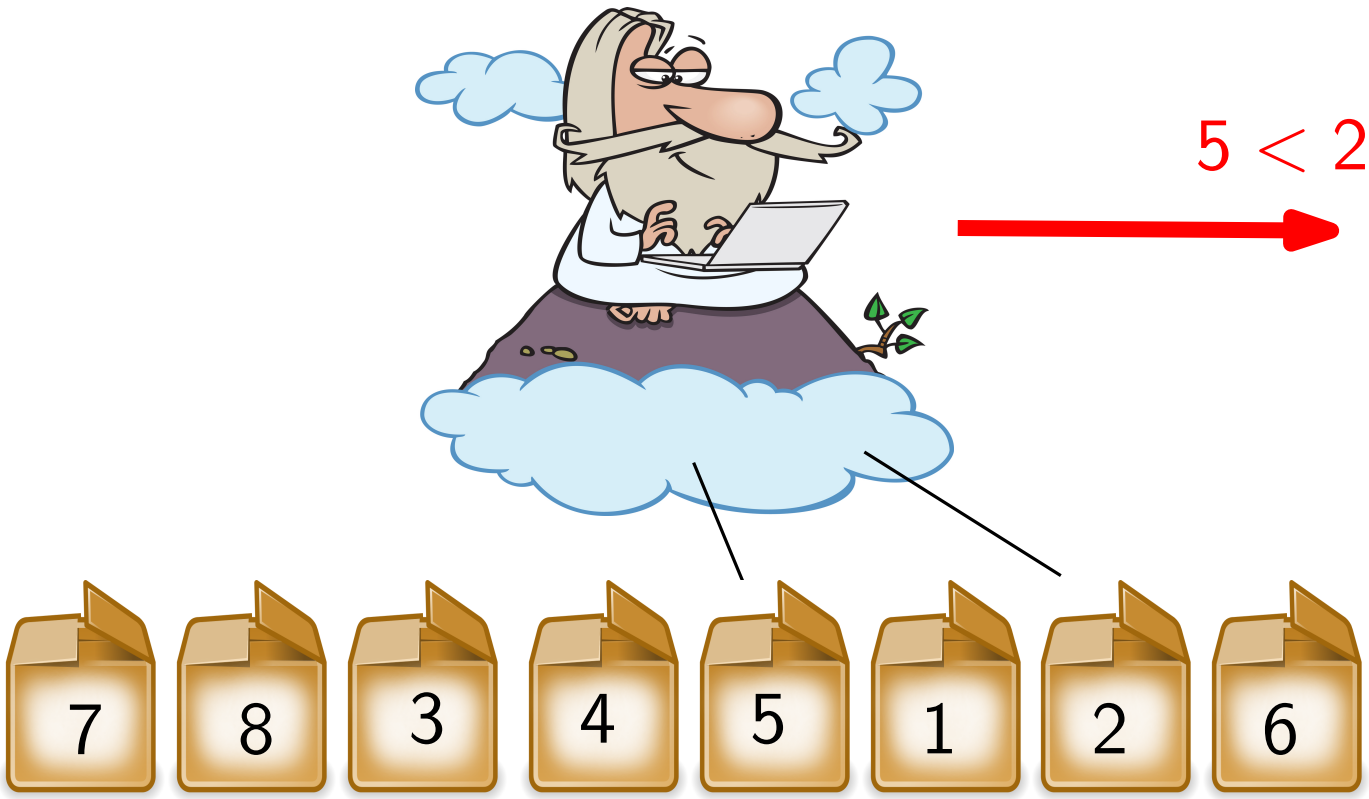
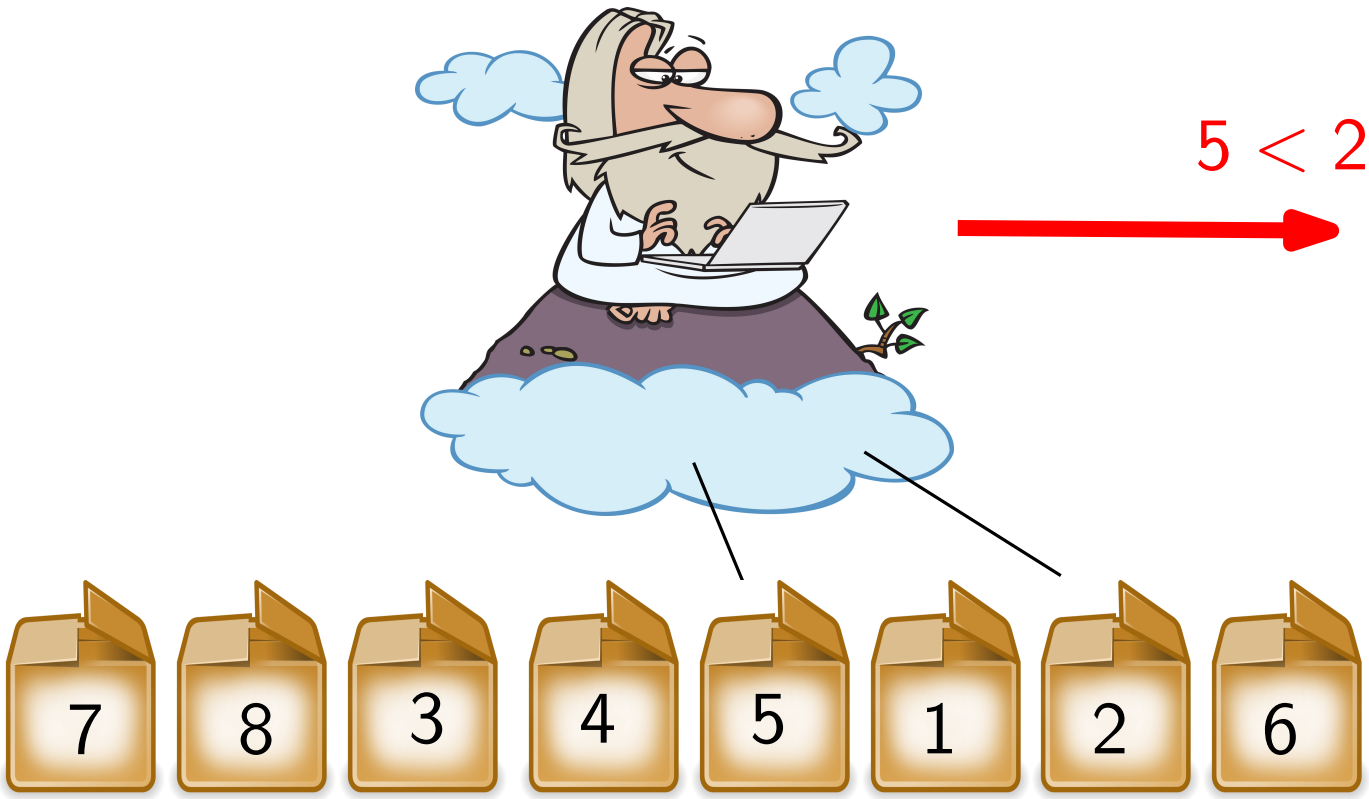


- error probability p constant

The Problem



- error probability p constant
- independent for each pair

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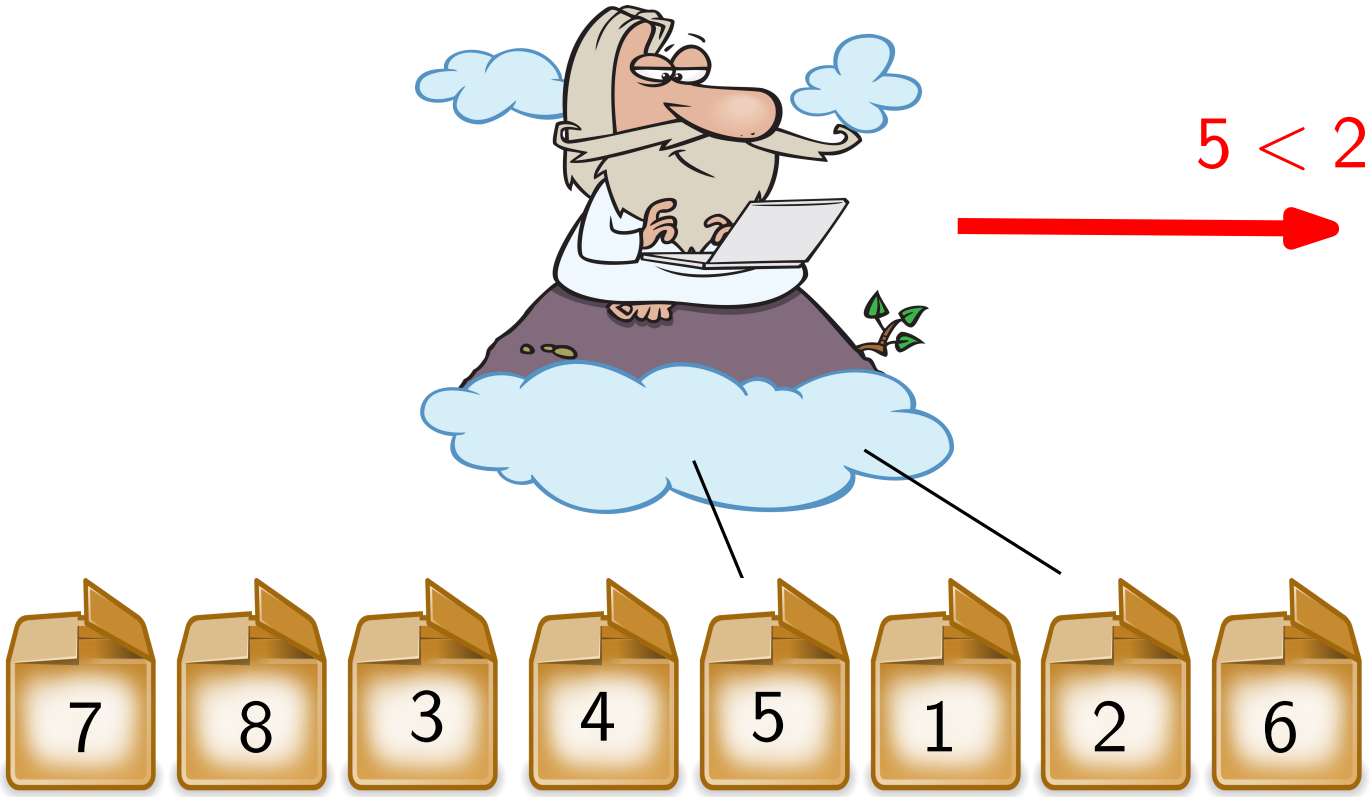

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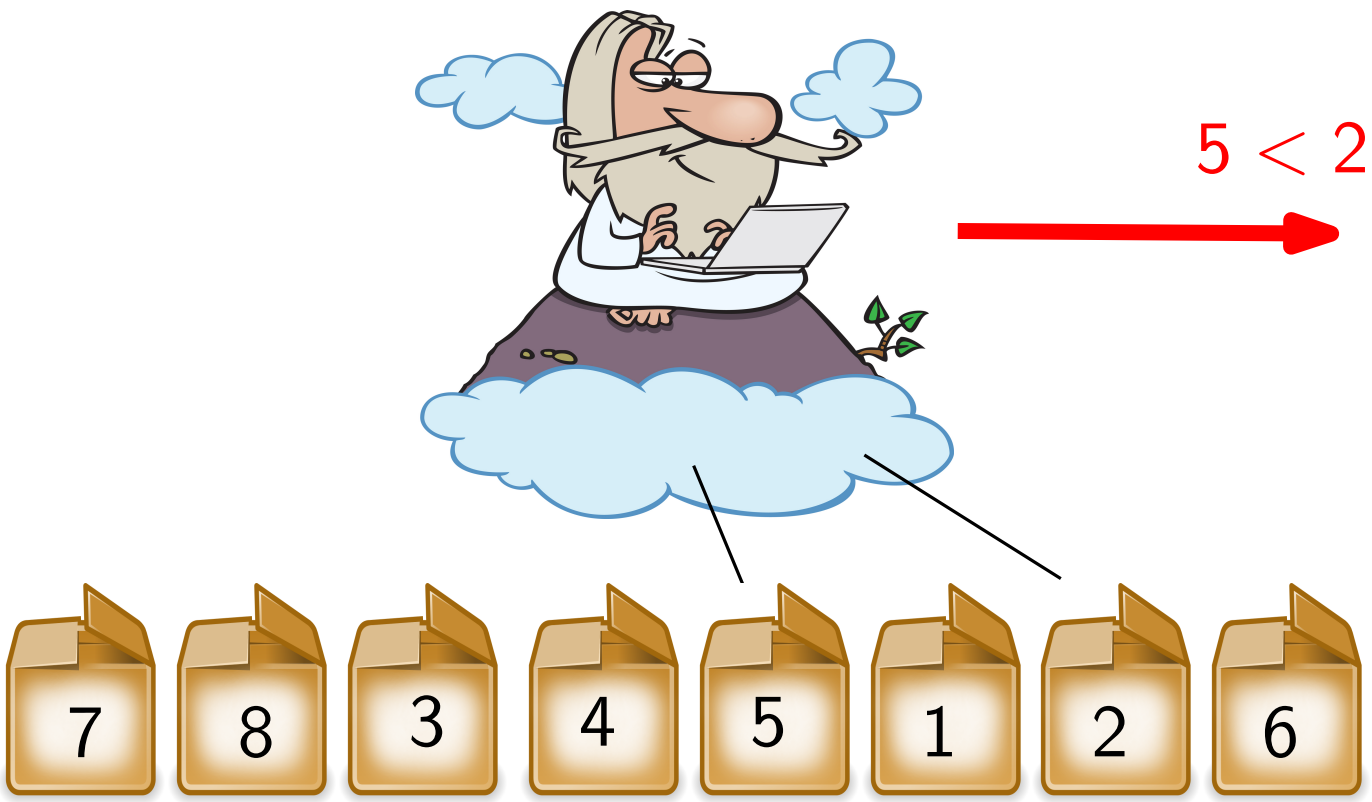
The Problem



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5%

The Problem

Repeating does not help

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- independent for each pair
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The Problem

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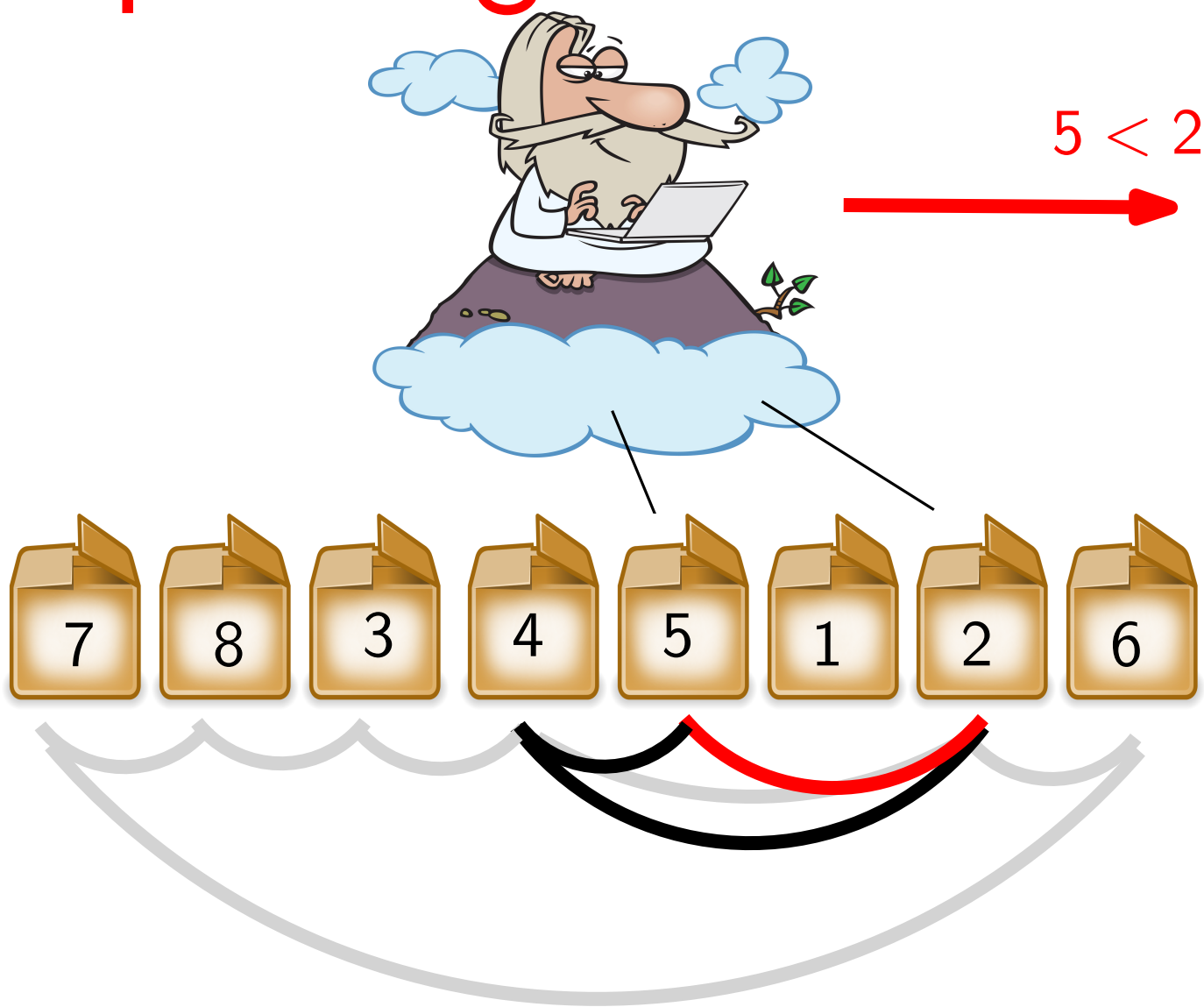
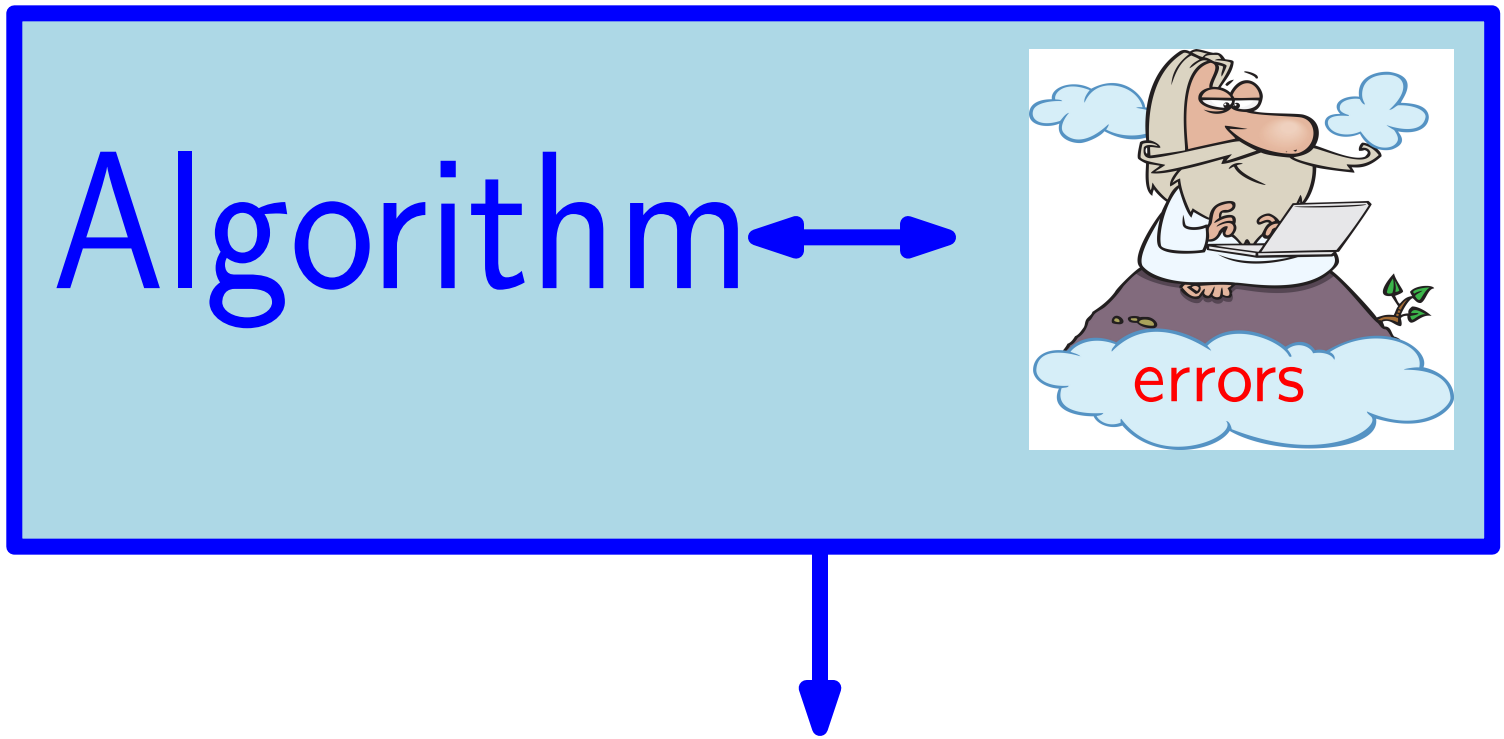


Can you sort?



Can you sort?

Algorithm

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

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Approx Sorted

Can you sort?

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Dislocation

What can be done?

Prior Results

Prior Results

MAX	**Dislocation**	TOTAL
$O(\log n)$		$O(n)$

Braverman & Mossel (SODA'08)

Prior Results

Time:	**Dislocation** MAX	TOTAL
$O(n^{3+C})$	$O(\log n)$	$O(n)$

Braverman & Mossel (SODA'08)

Prior Results

Time:	Dislocation	
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$$\frac{110525}{(1/2-p)^4}$$

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Subquadratic time?

Our Contribution

YES

Our Contribution

YES

Time:	Dislocation	
	MAX	TOTAL
$O(n^{3/2})$	$O(\log n)$	$O(n)$

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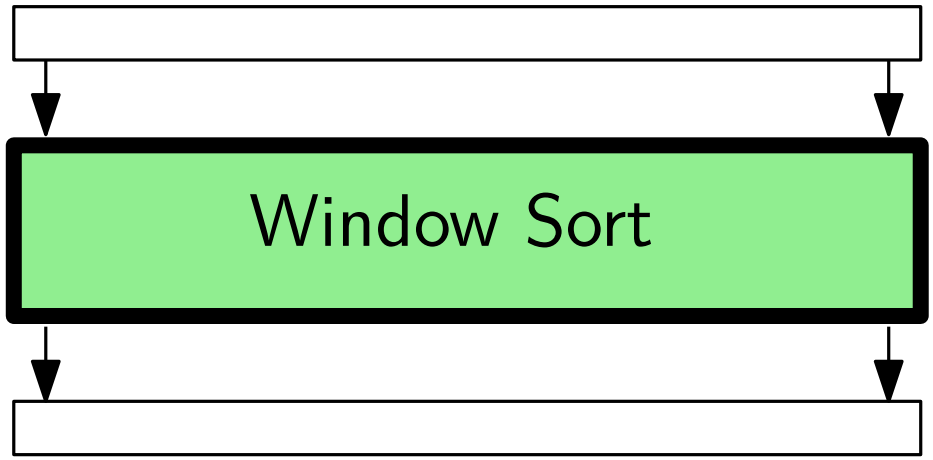
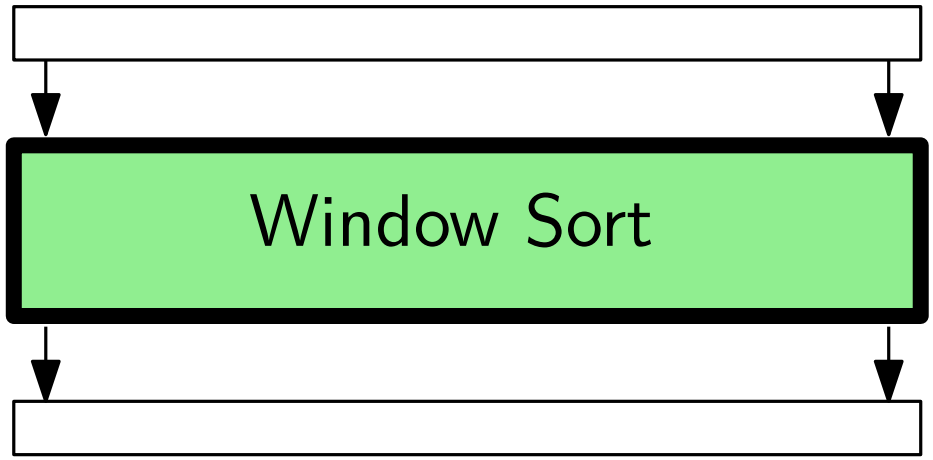
randomized algorithm → "derandomized" algorithm

$O(n^2)$-Time Algorithm



Geissmann, Leucci, Liu, Penna (ISAAC'17)

$O(n^2)$-Time Algorithm



errors well-spread $\Rightarrow$ success

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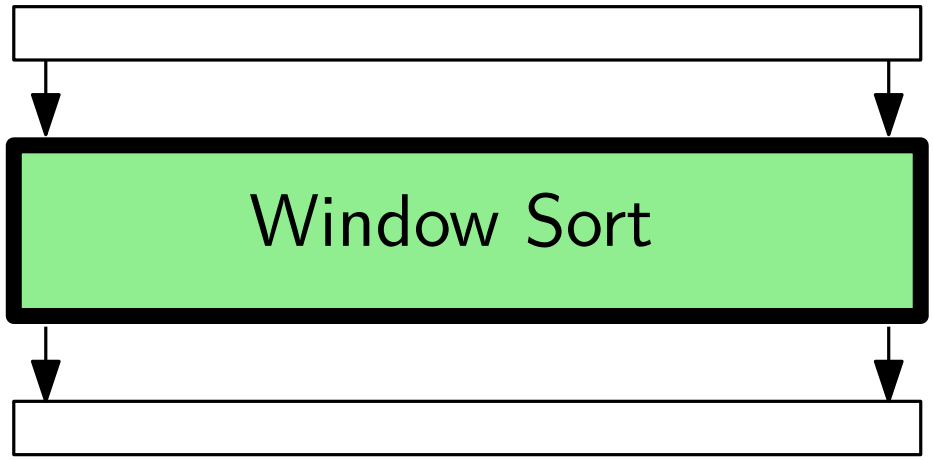

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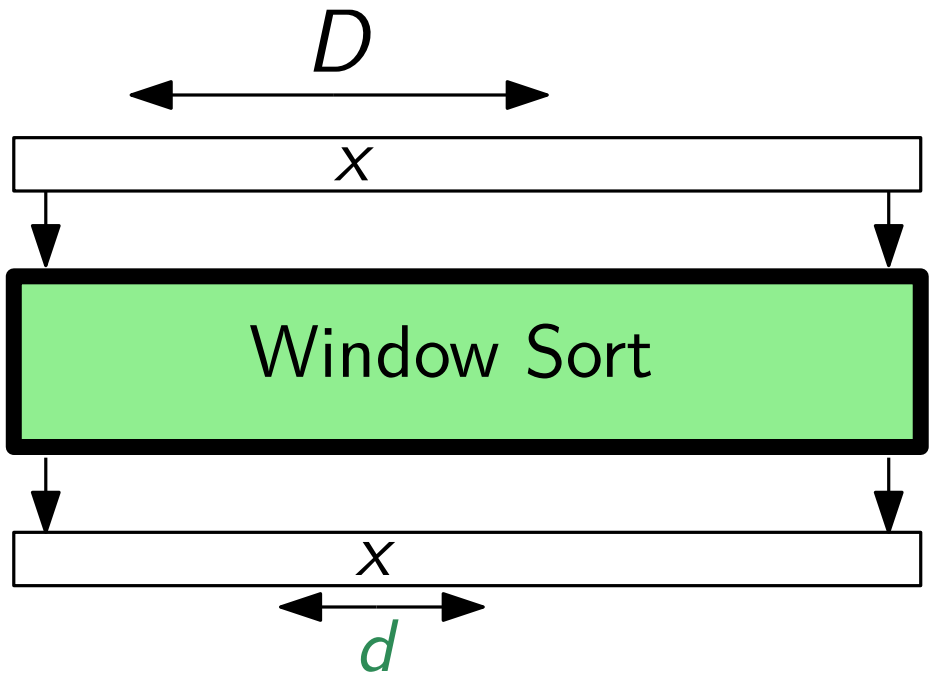

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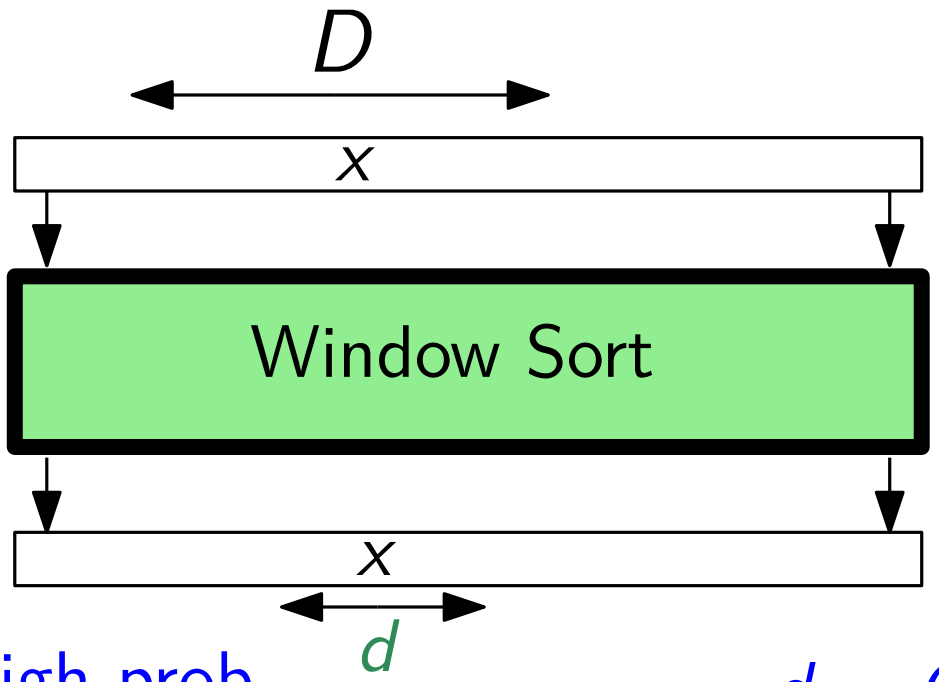

with high prob $d = O(\log n)$

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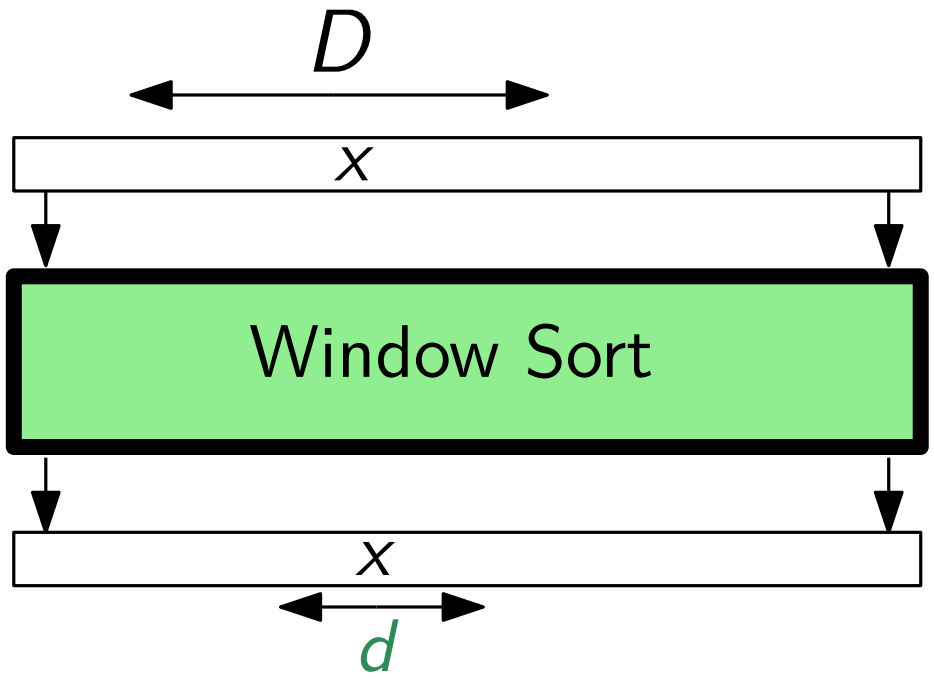

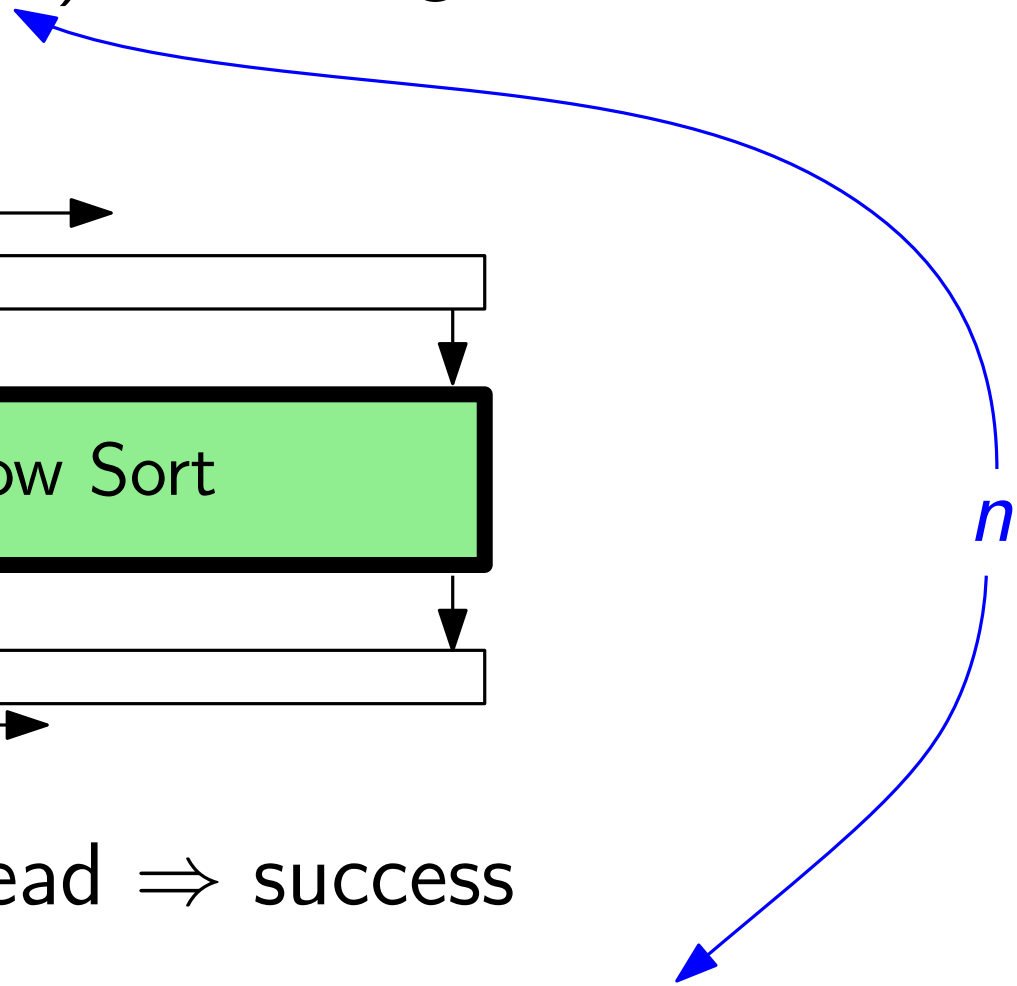

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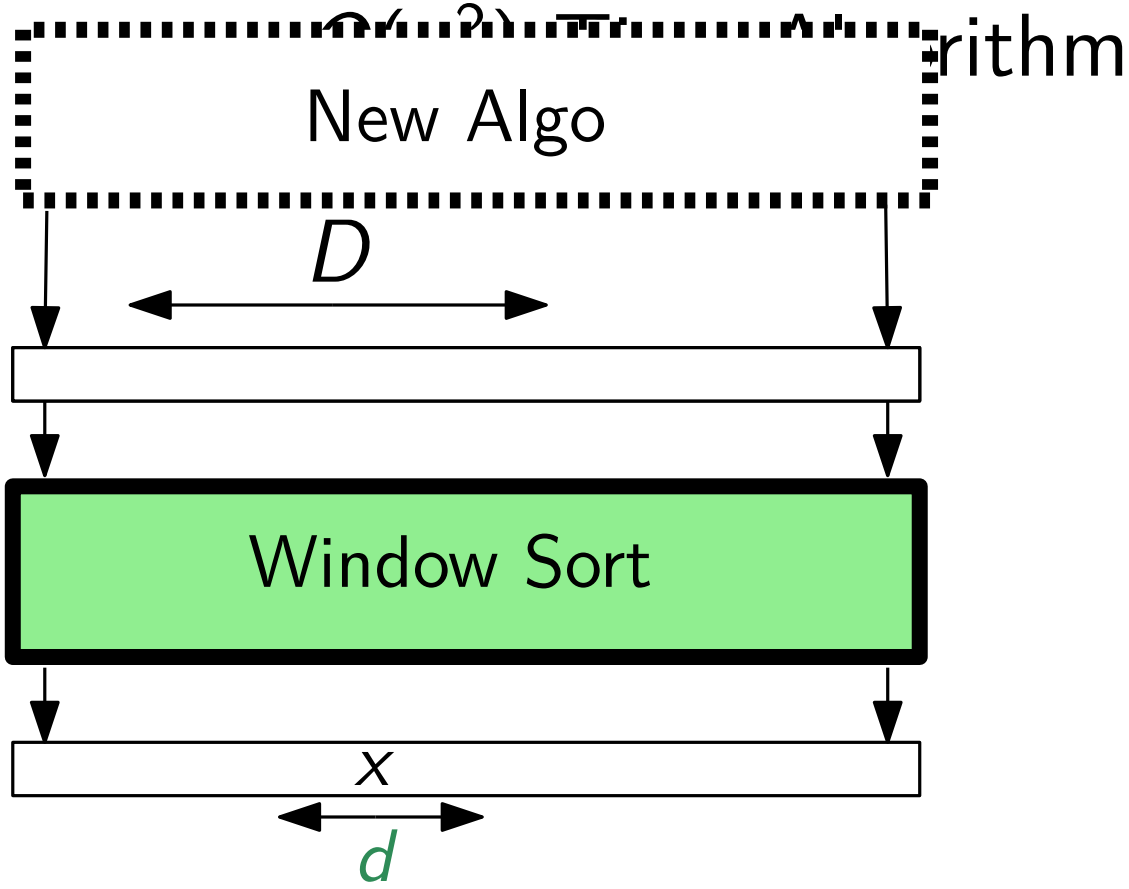

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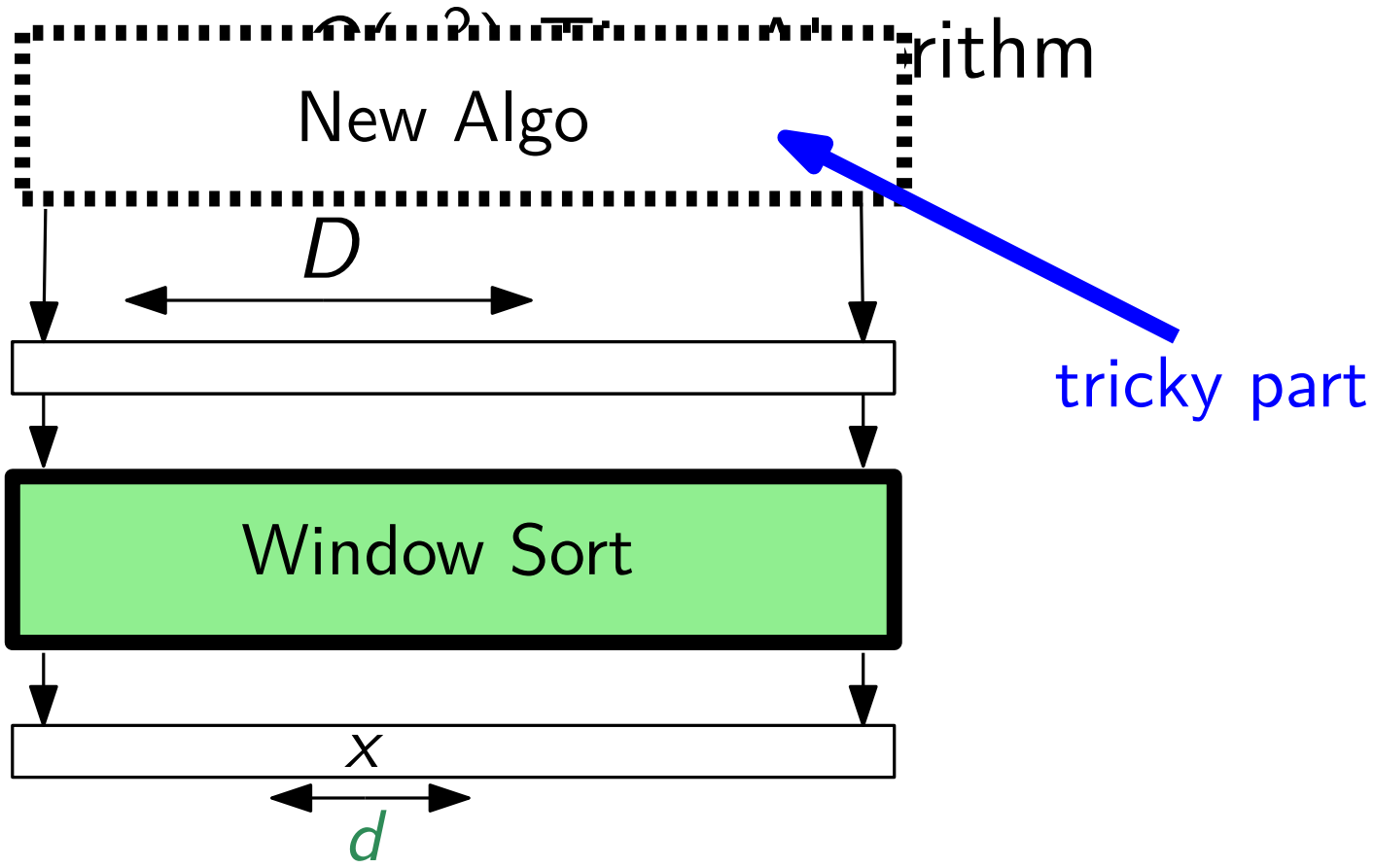

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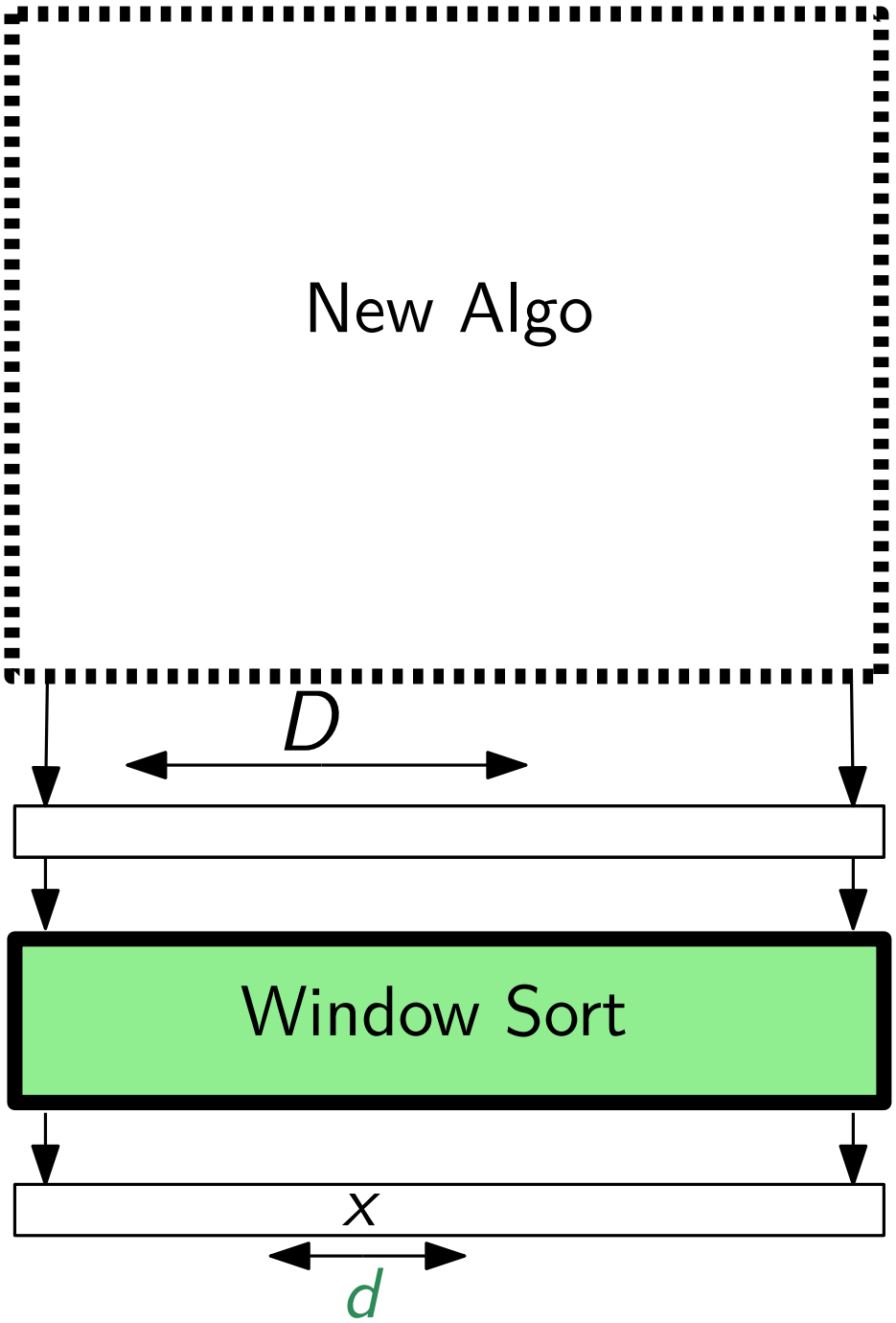
Simple Faster Algo



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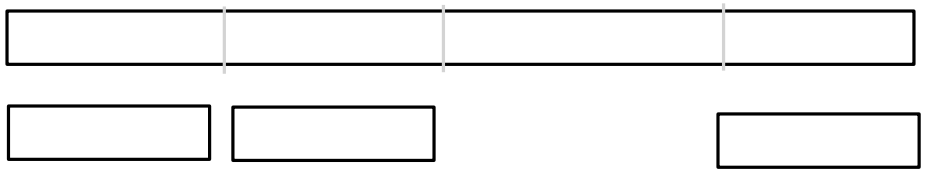

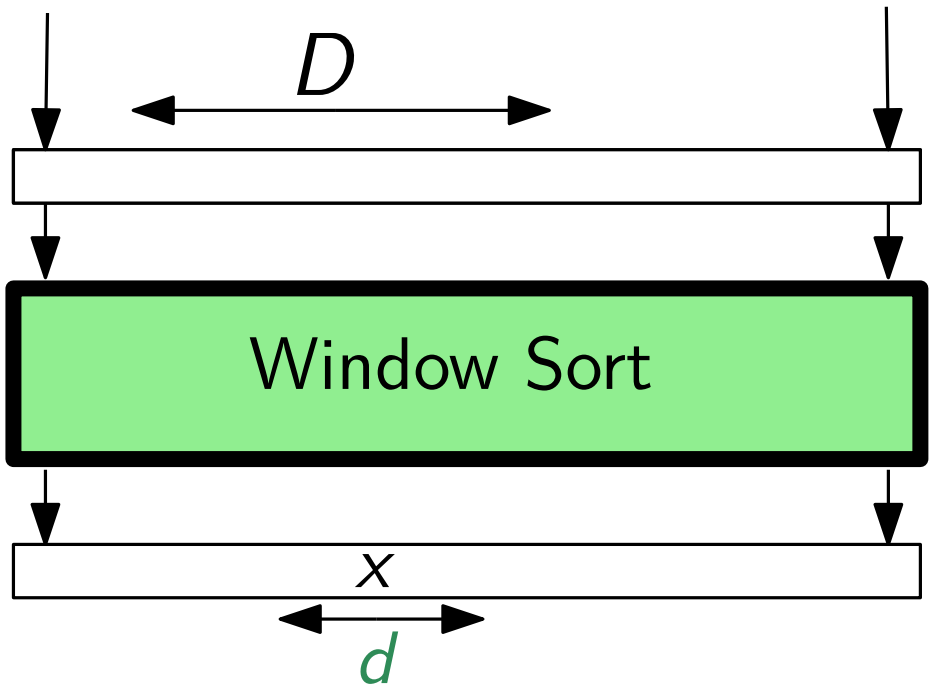

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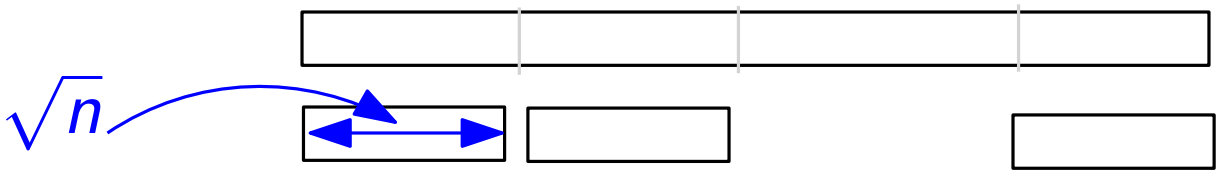

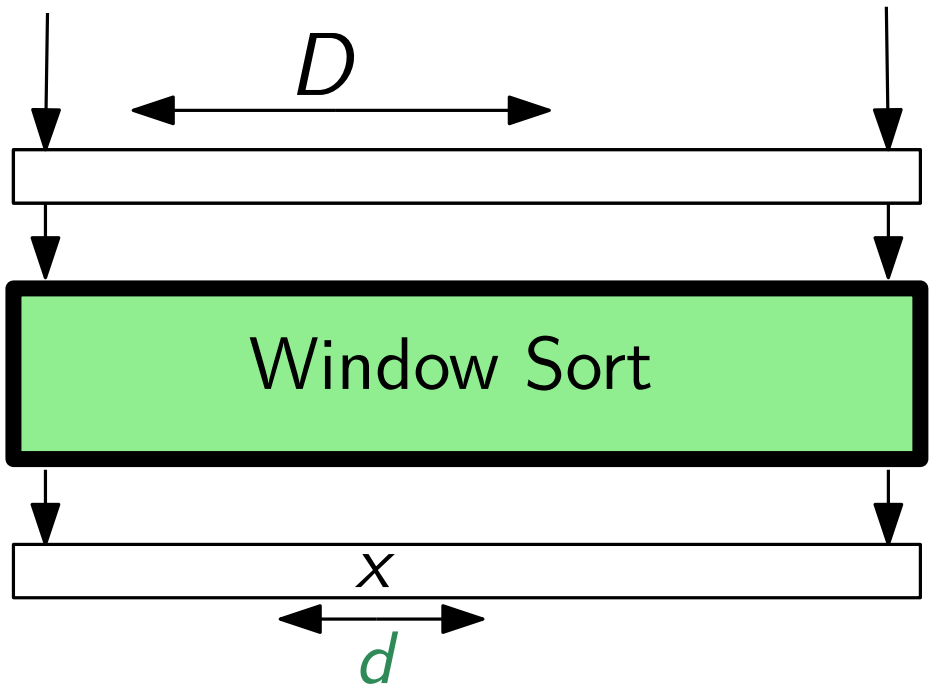

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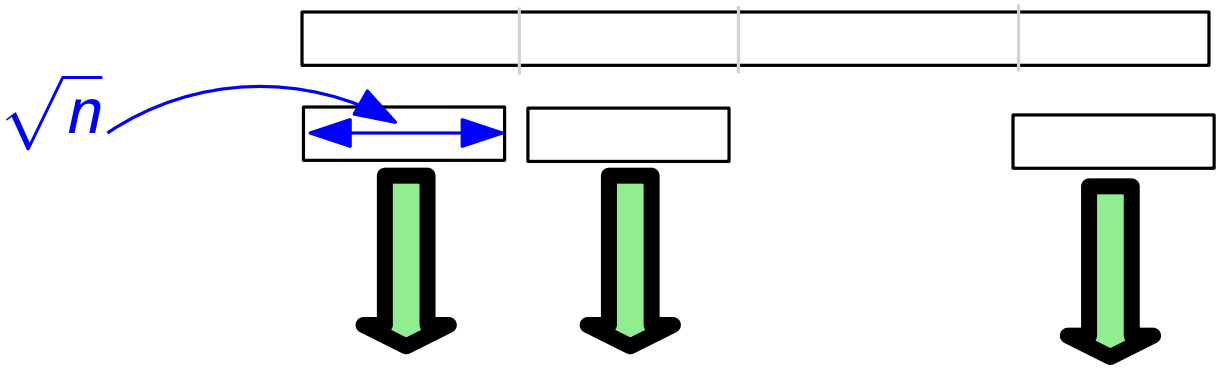

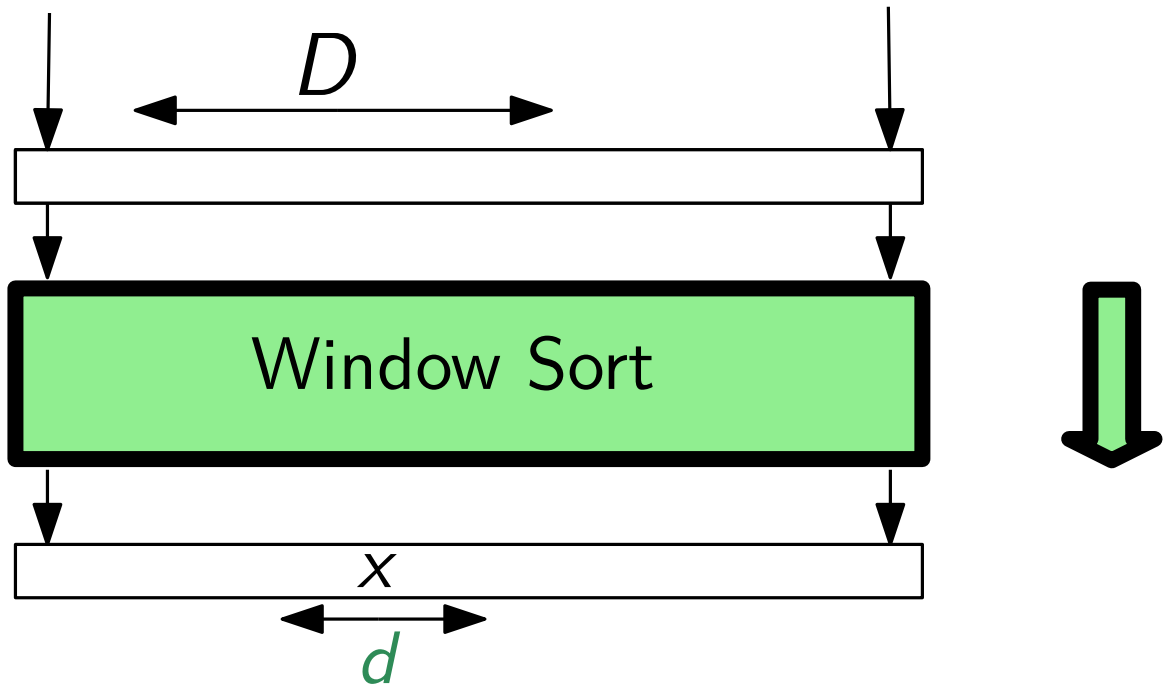

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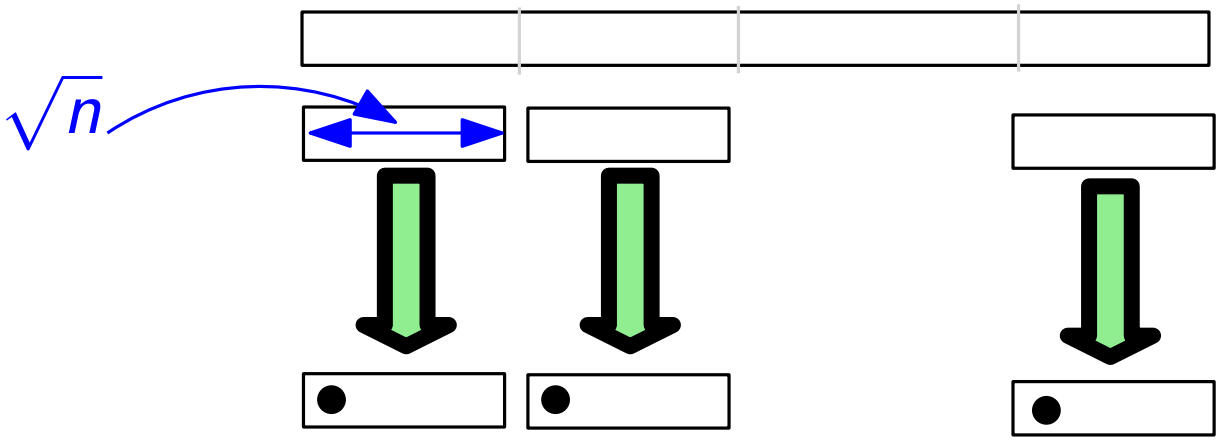

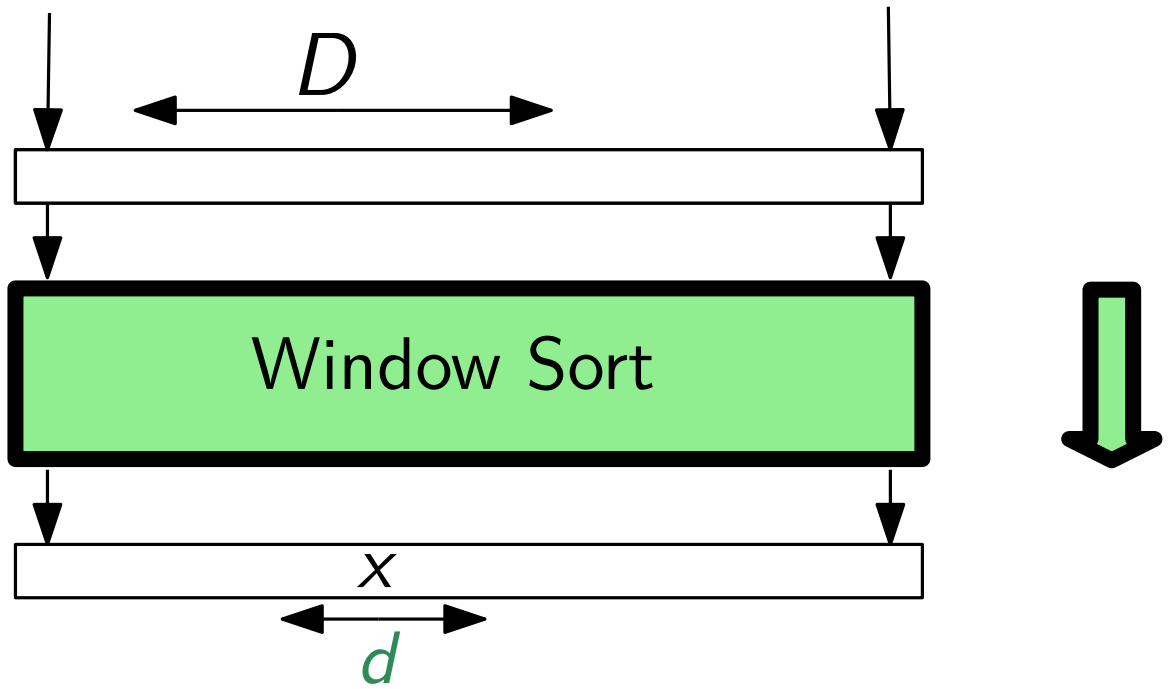

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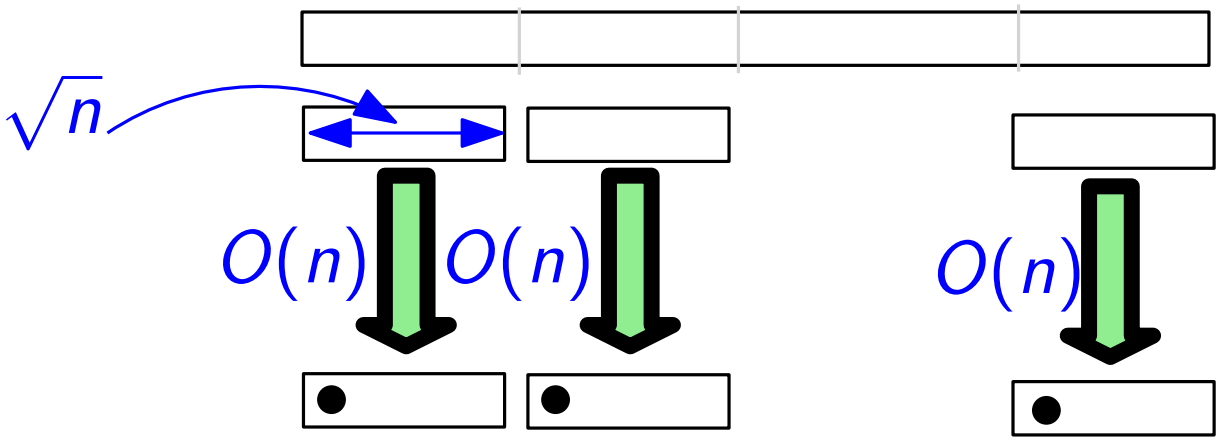

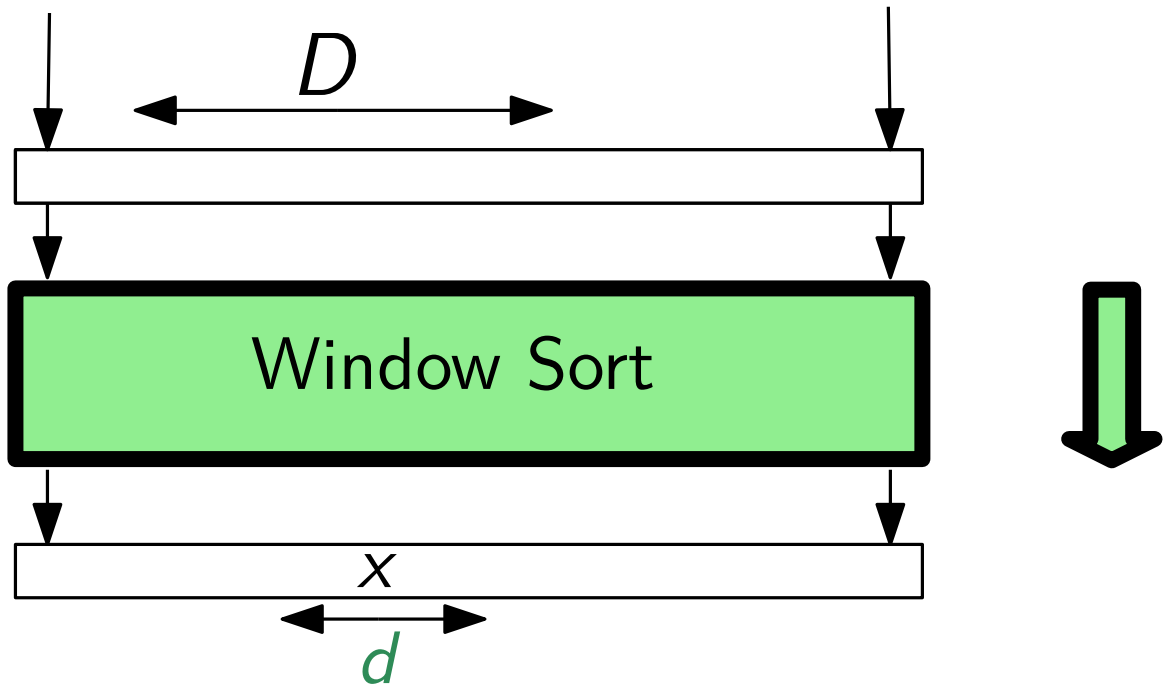

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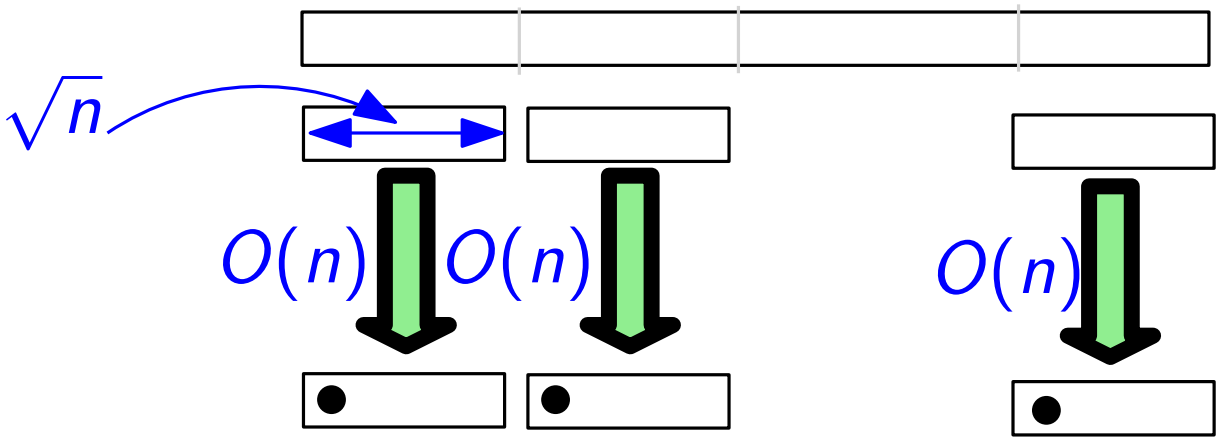
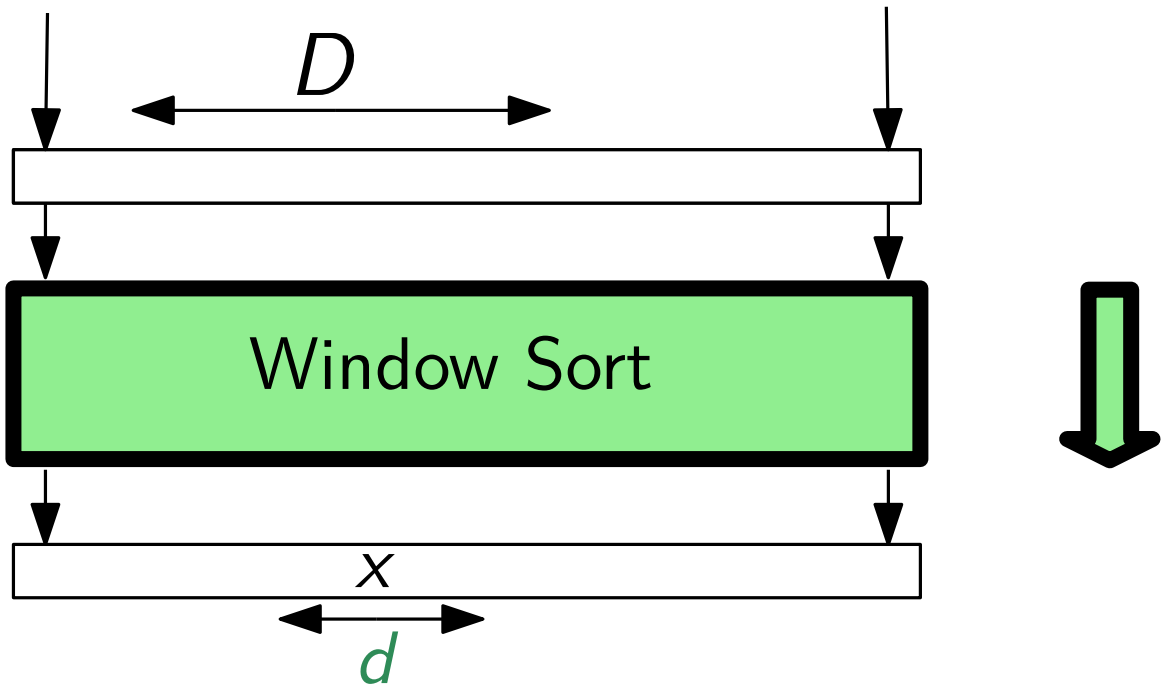


$O(n^{3/2})$



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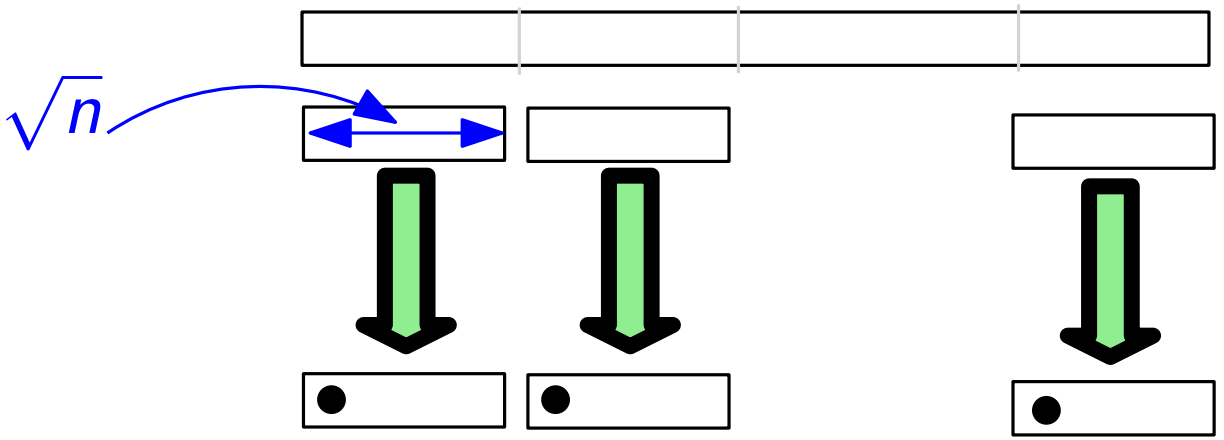

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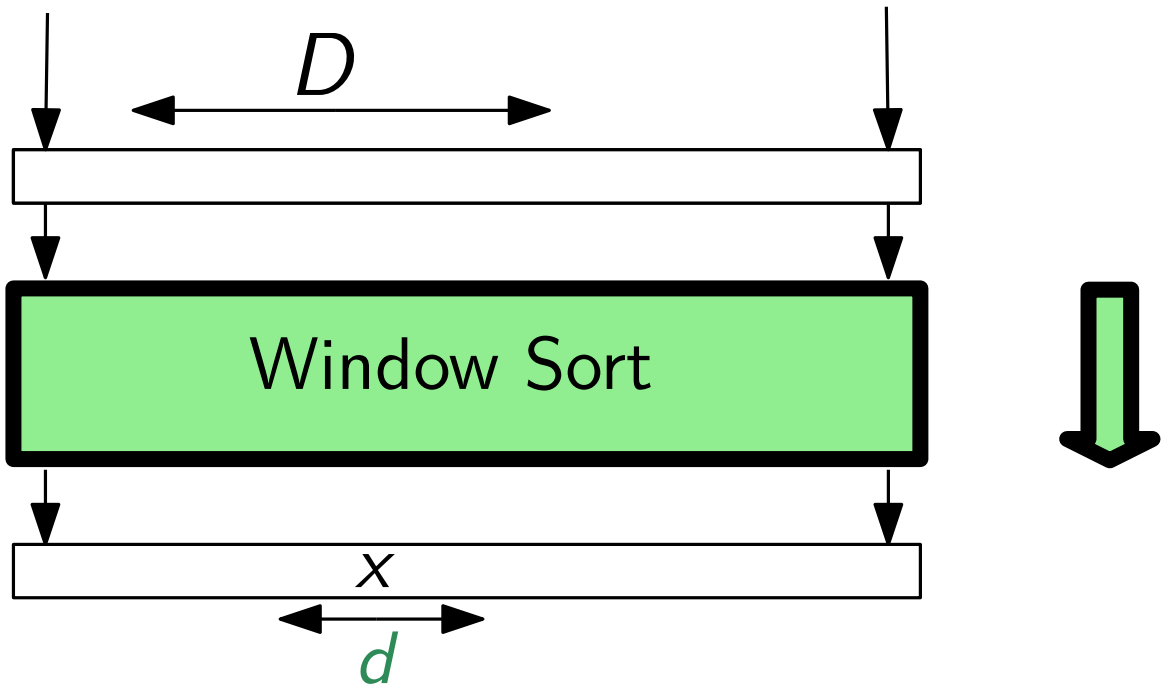
NOT ENOUGH!



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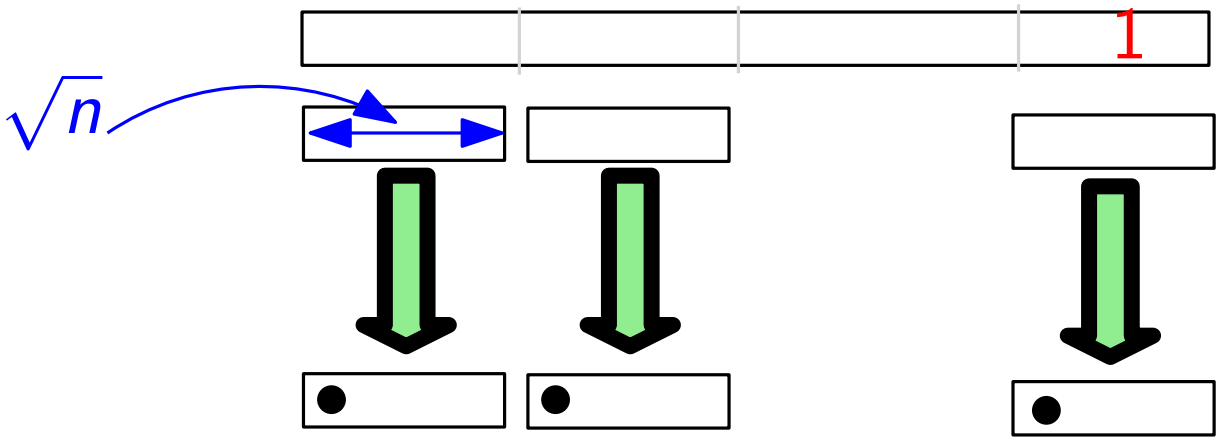

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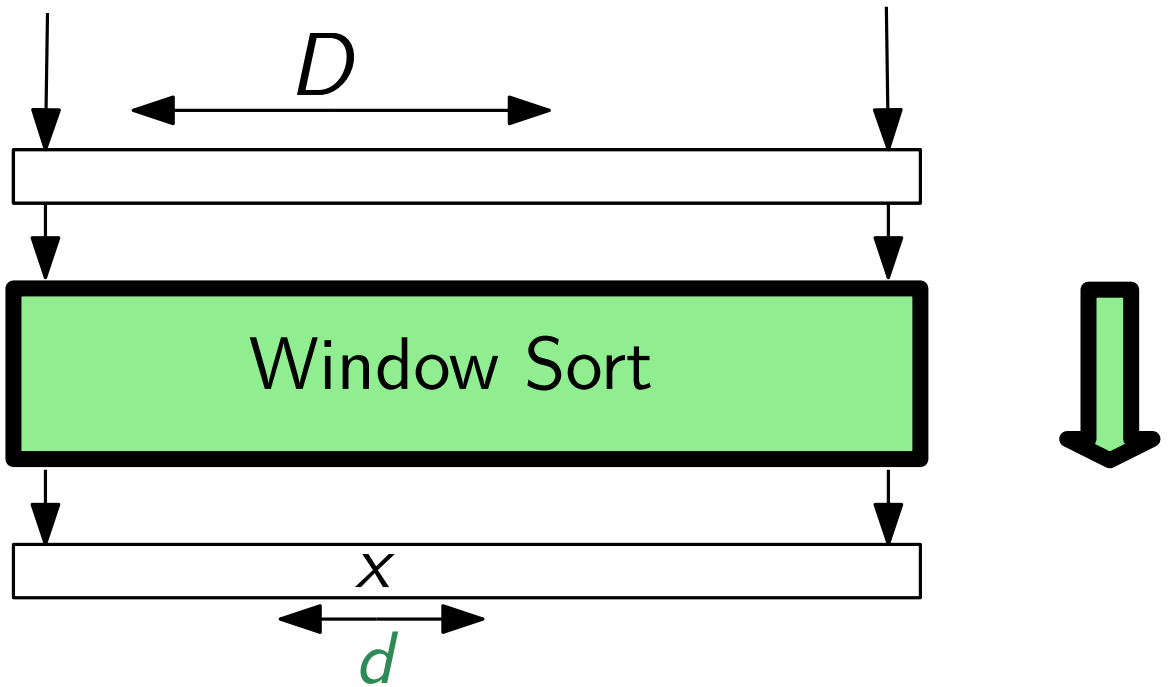

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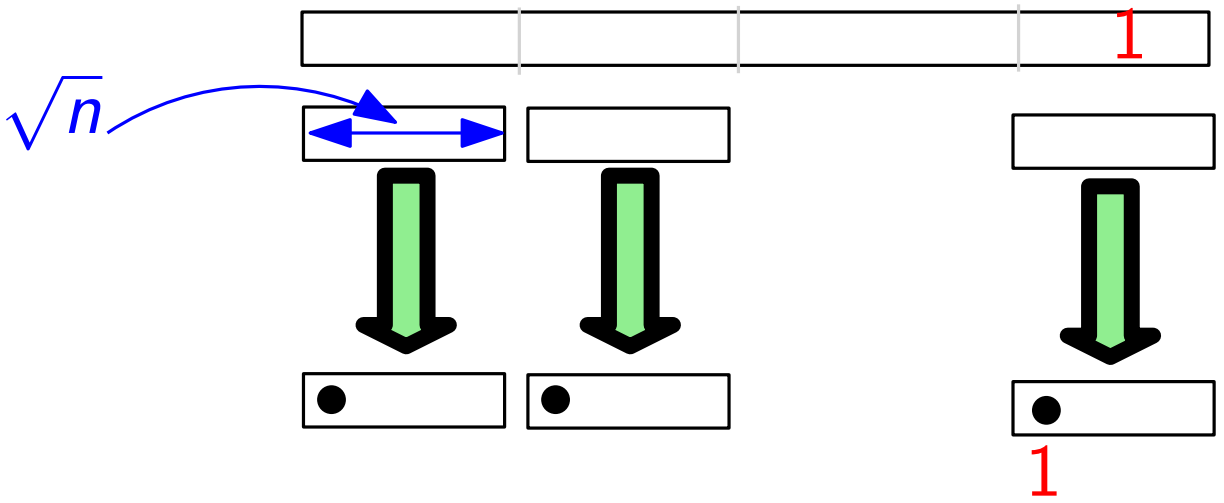

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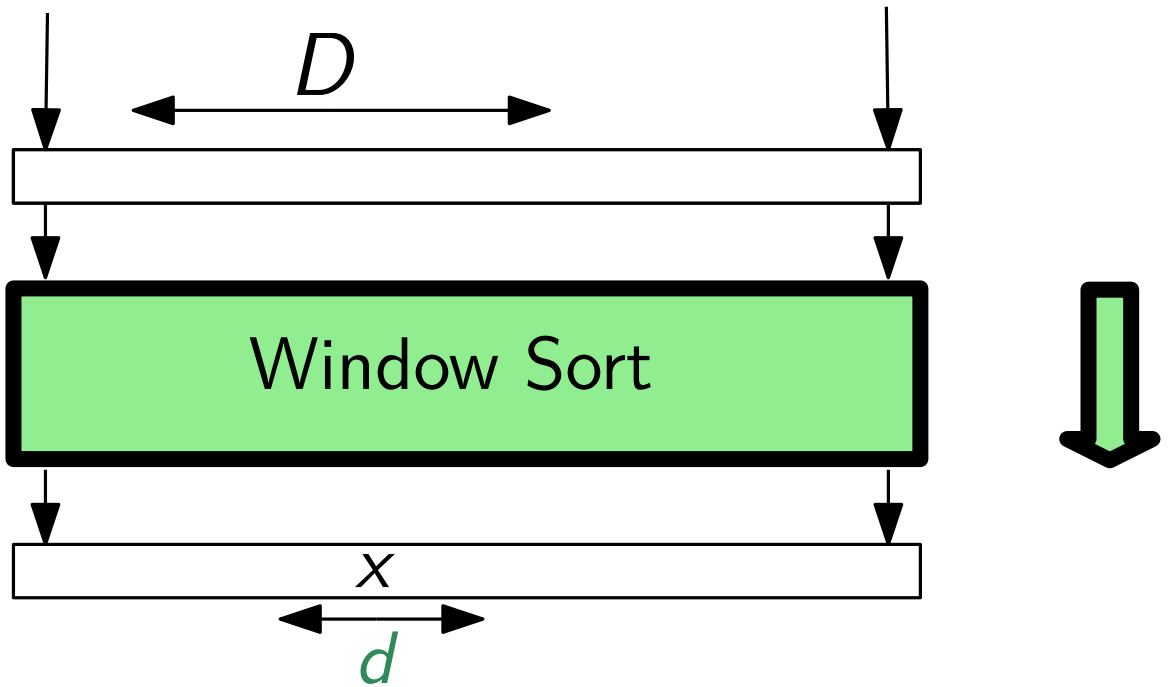

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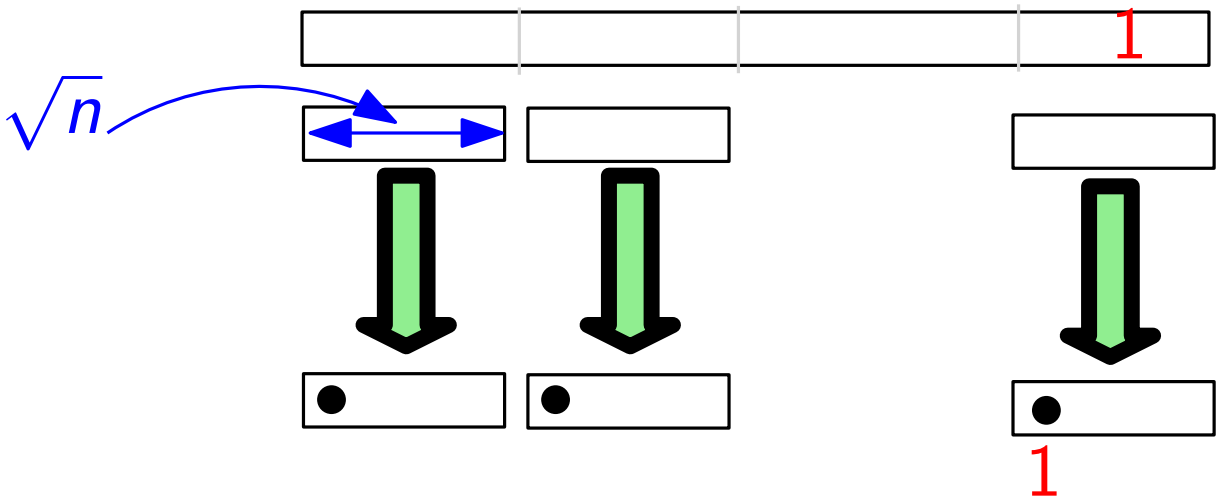

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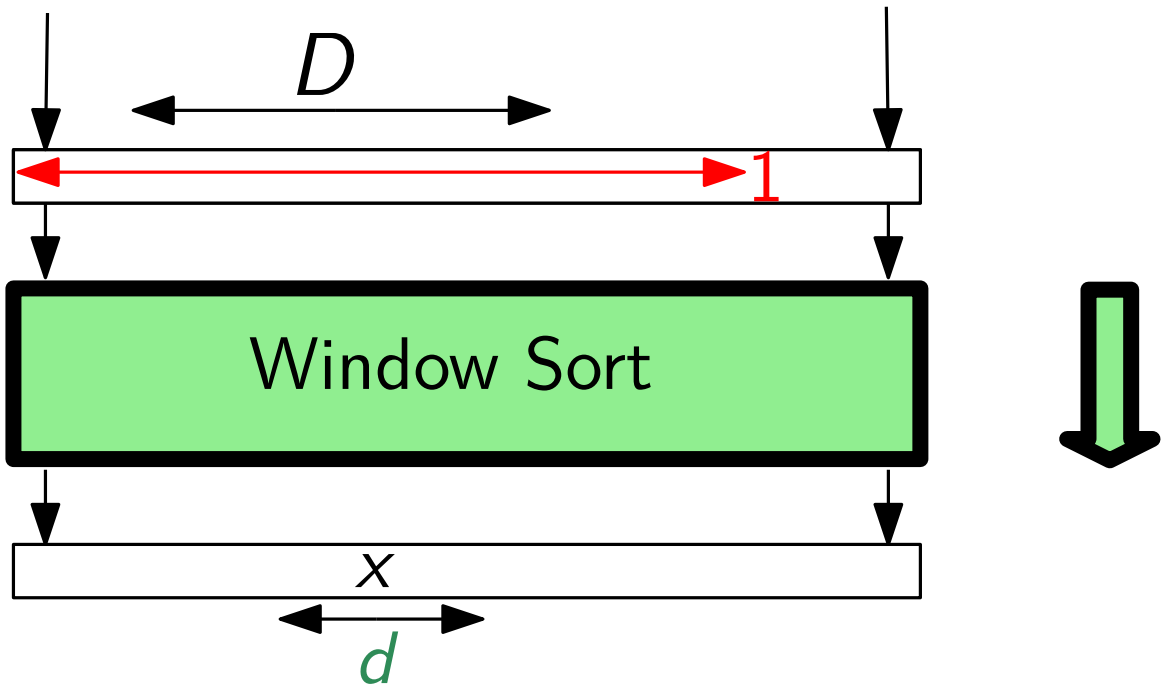

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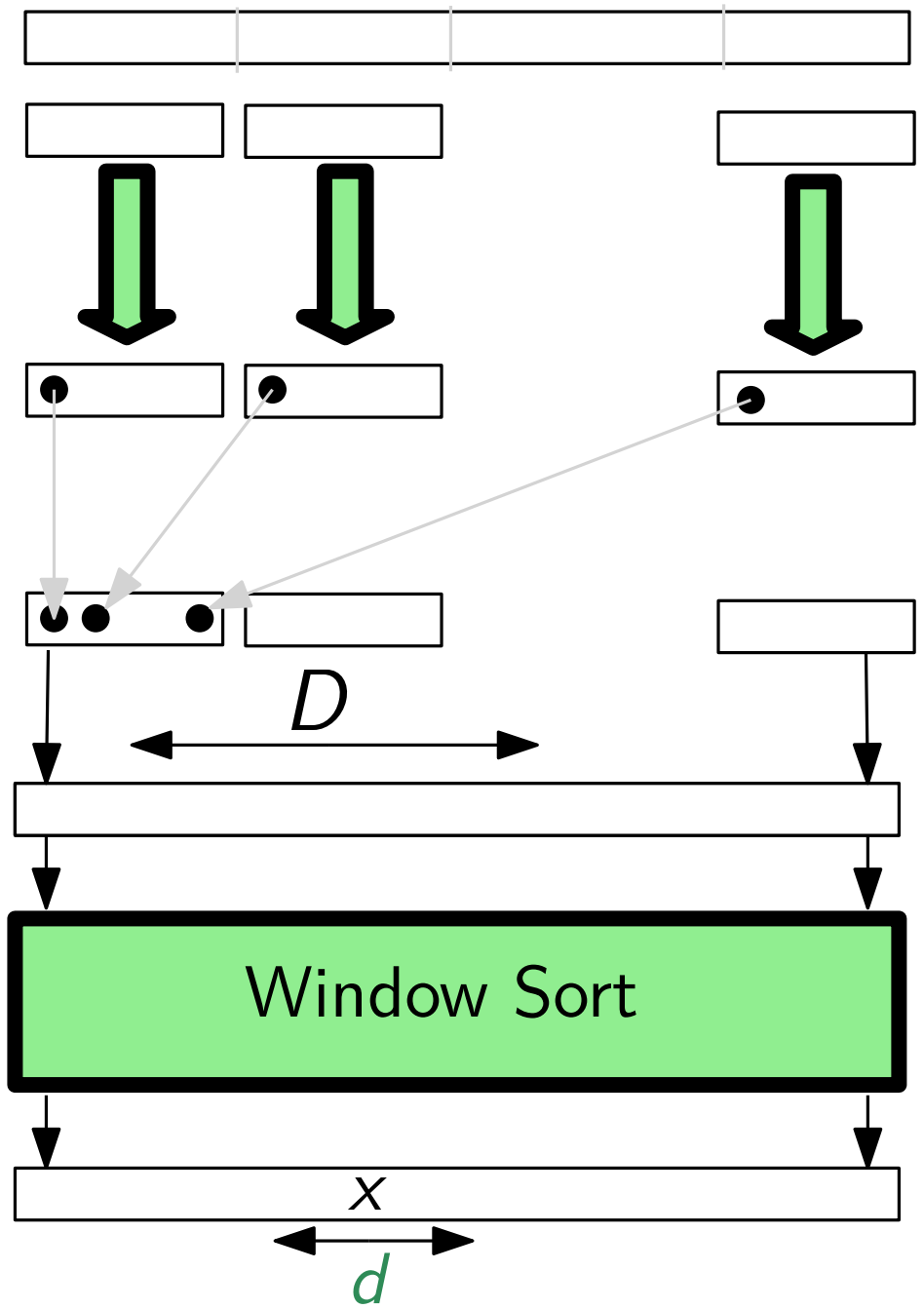
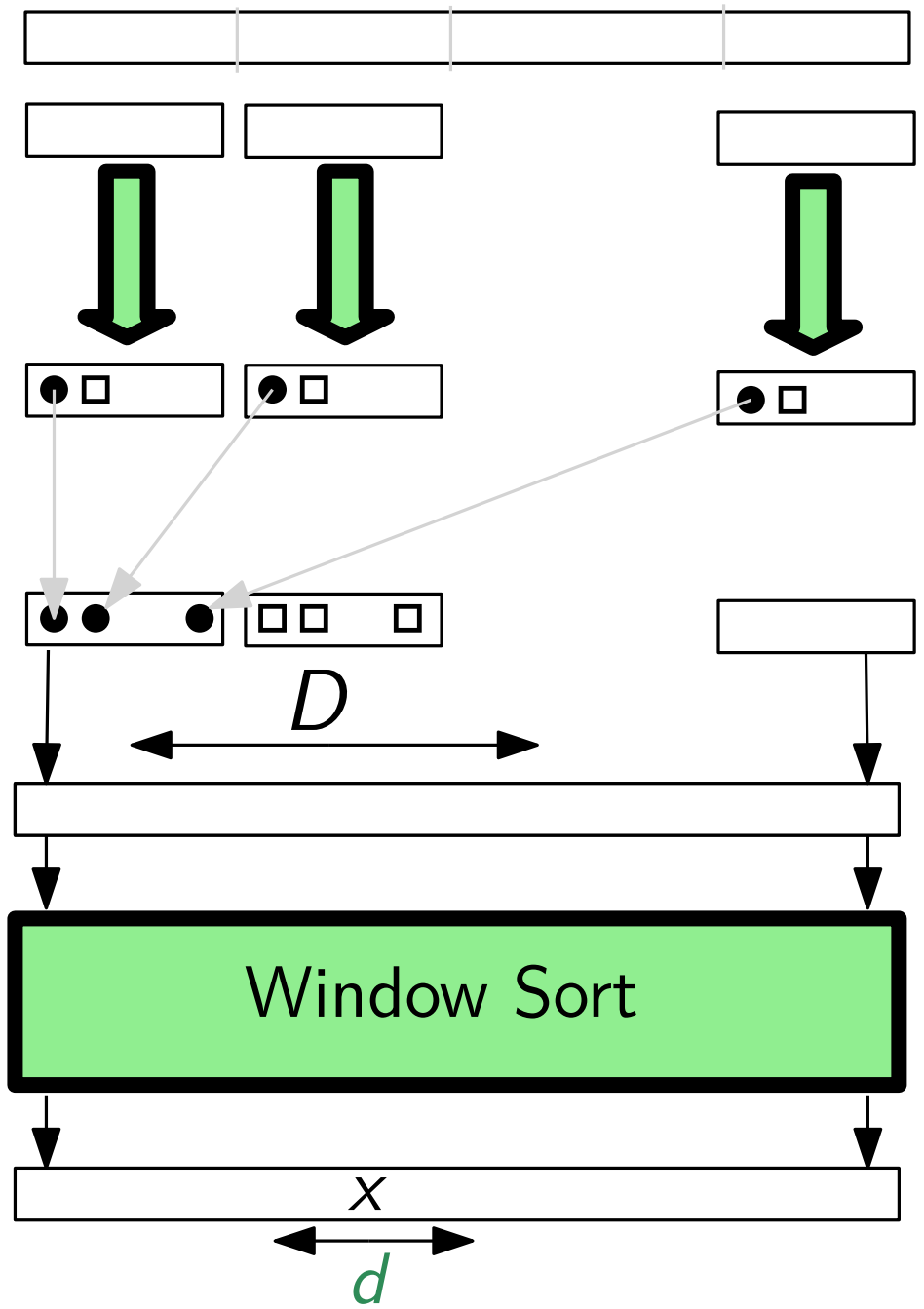


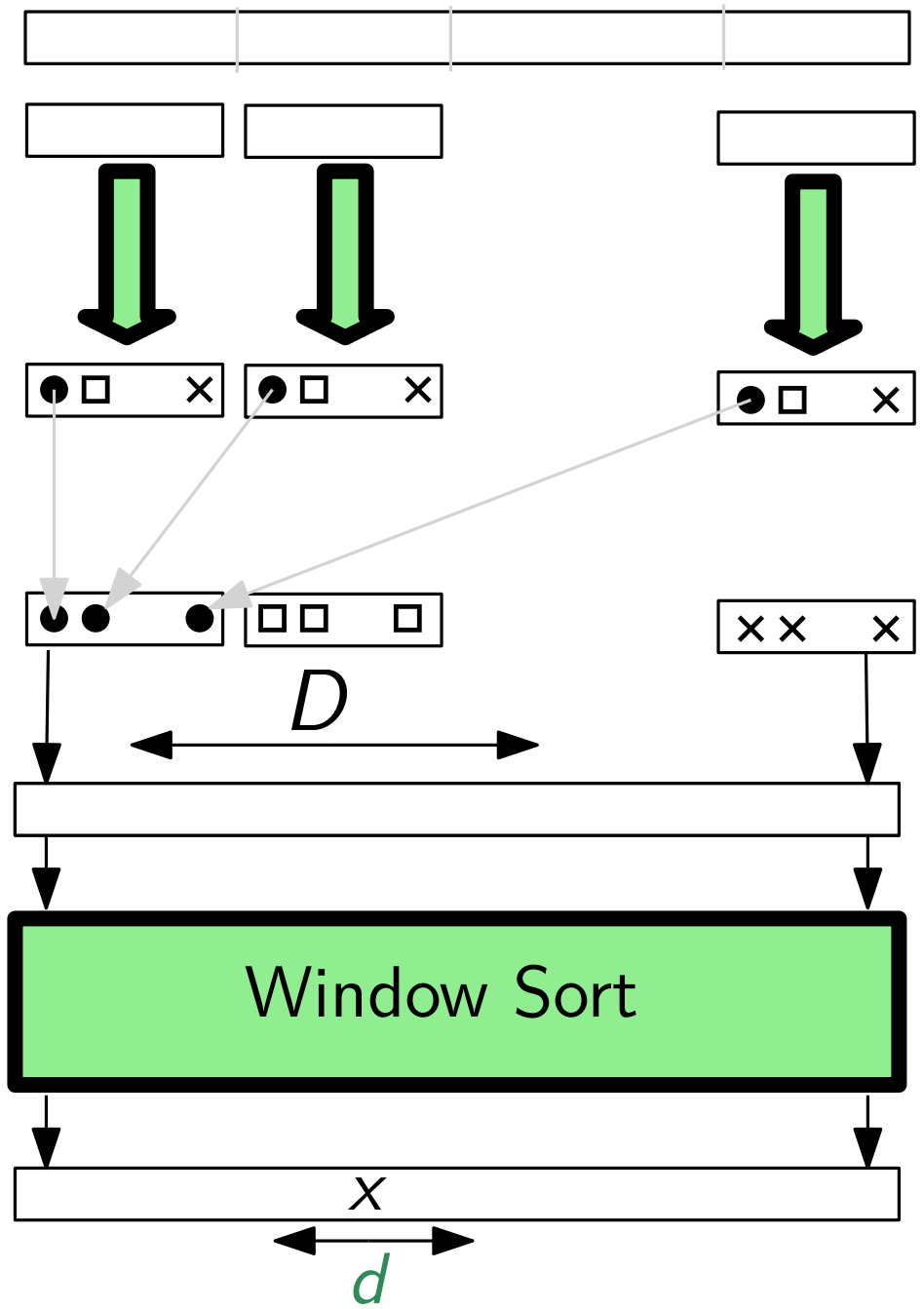
$O(n^{3/2})$

Simple Faster Algo



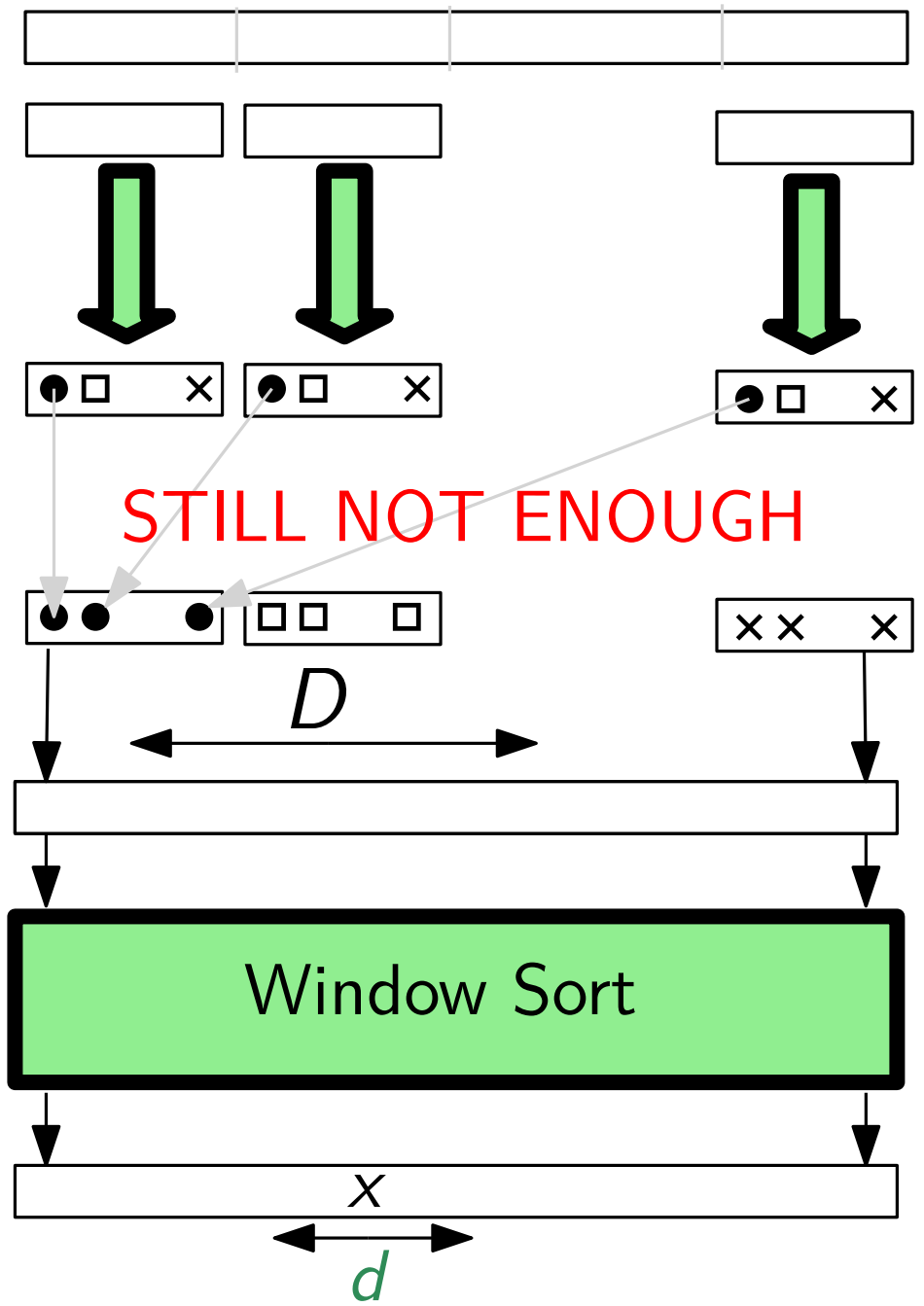
$O(n^{3/2})$

Simple Faster Algo



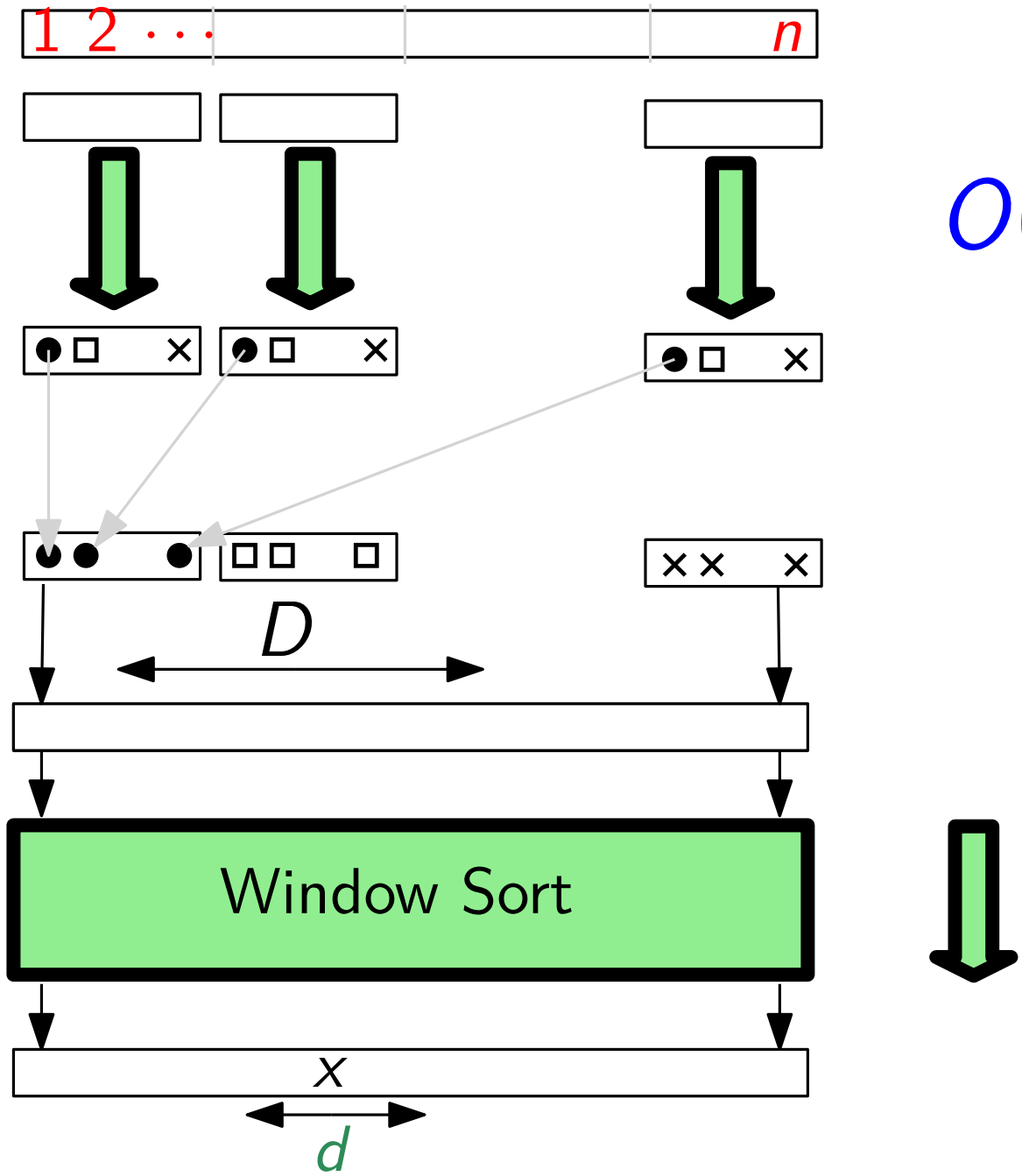
$O(n^{3/2})$

Simple Faster Algo



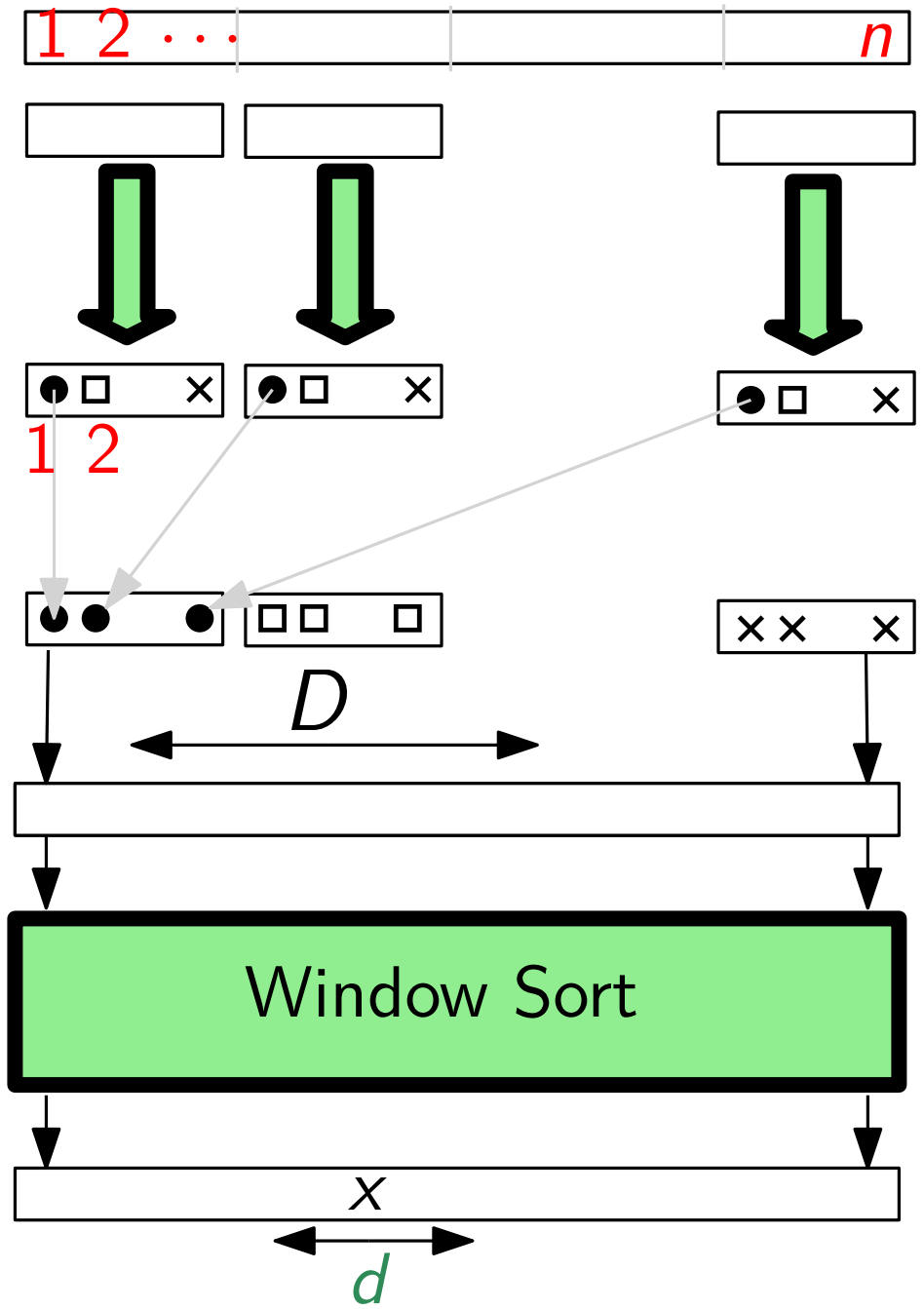
$O(n^{3/2})$

Simple Faster Algo



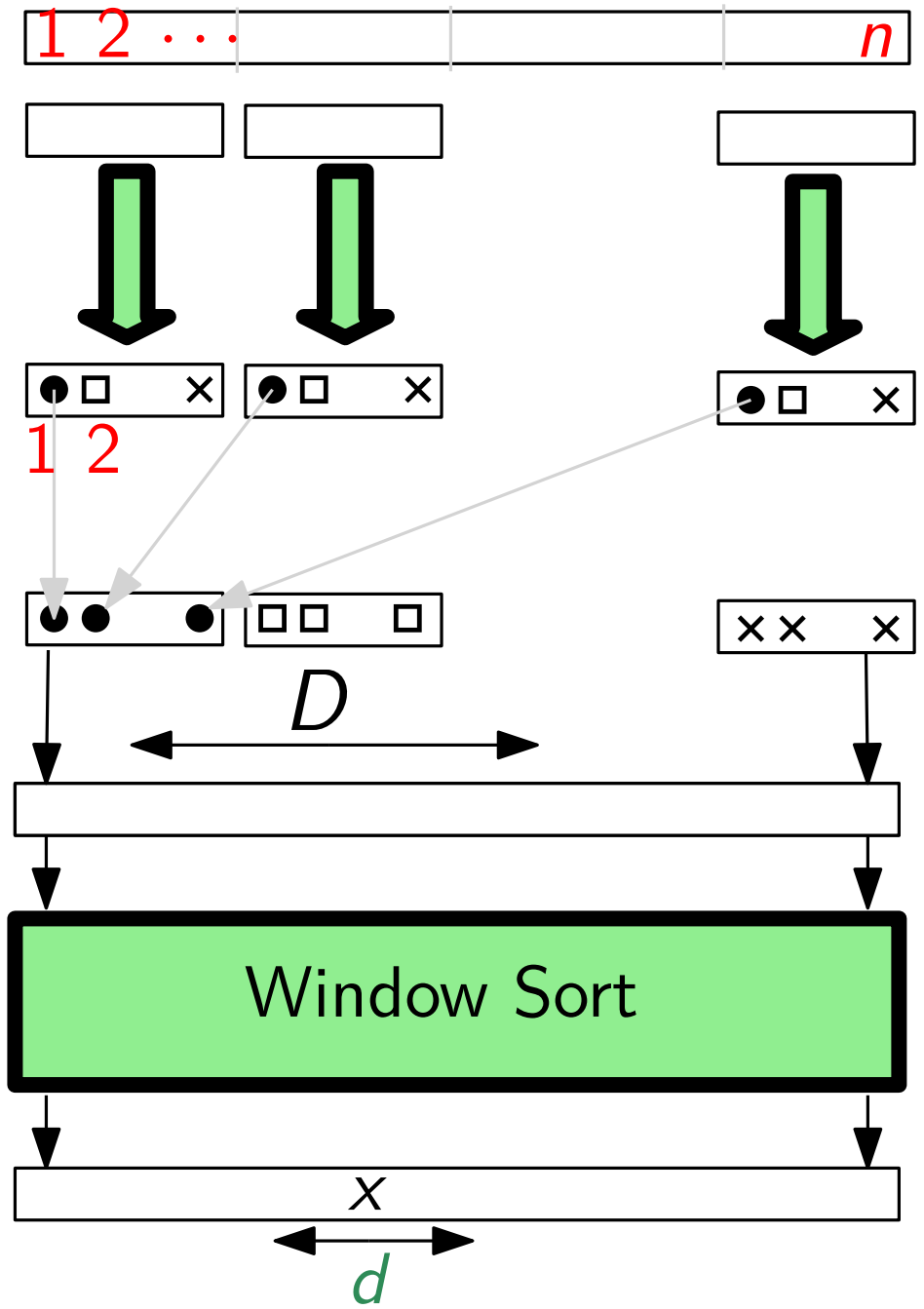
$O(n^{3/2})$

Simple Faster Algo



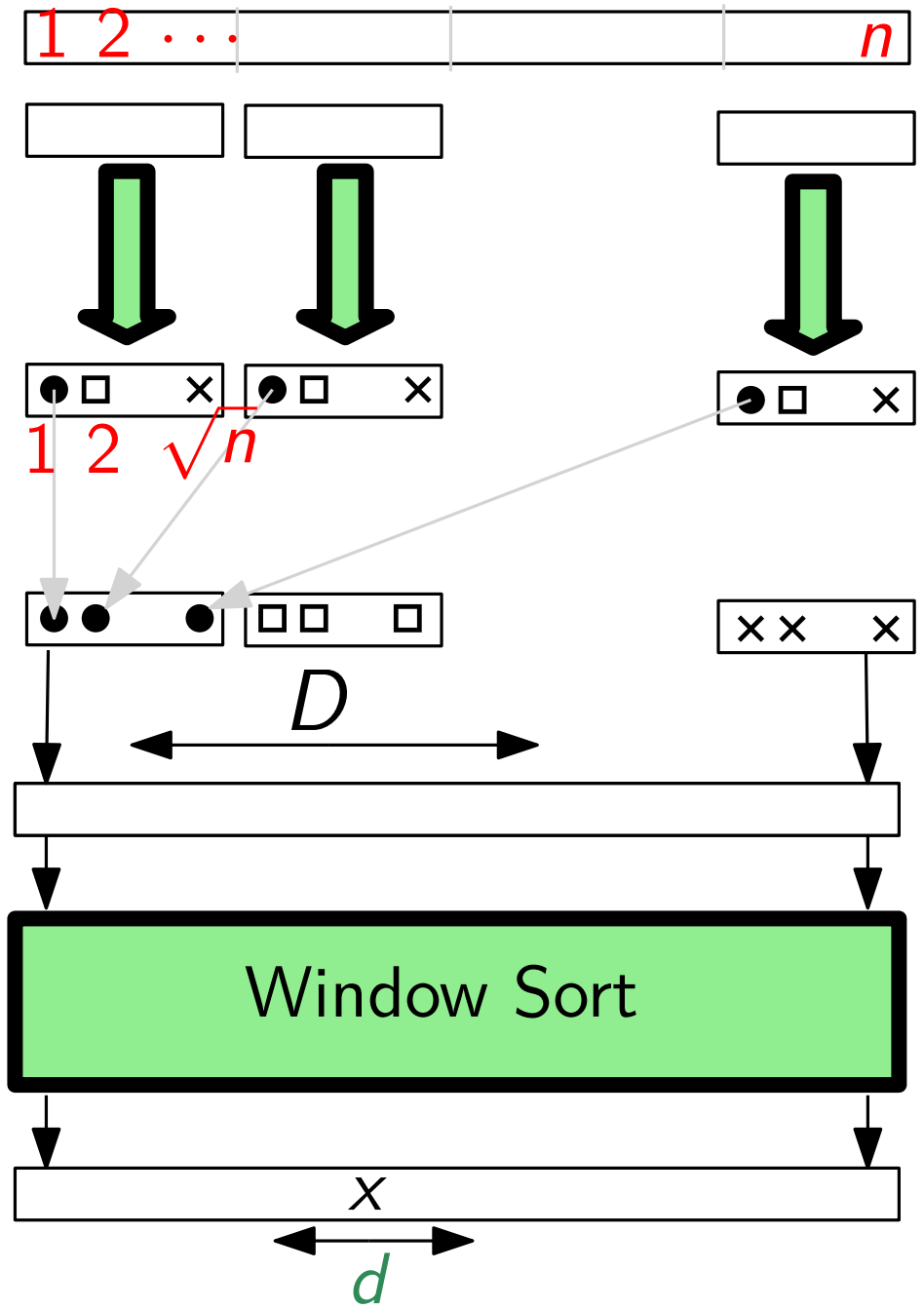
$O(n^{3/2})$

Simple Faster Algo



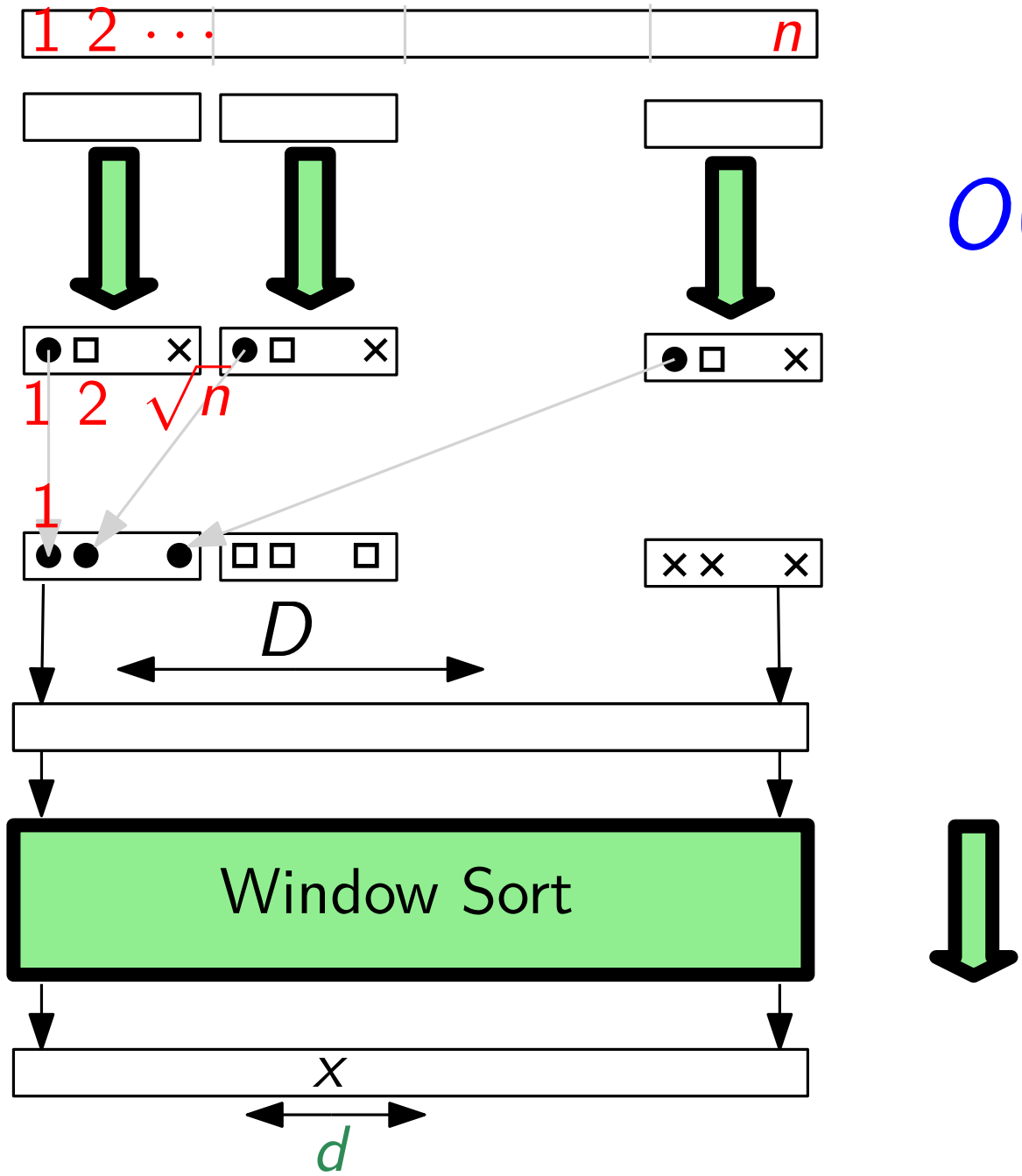
$O(n^{3/2})$

Simple Faster Algo



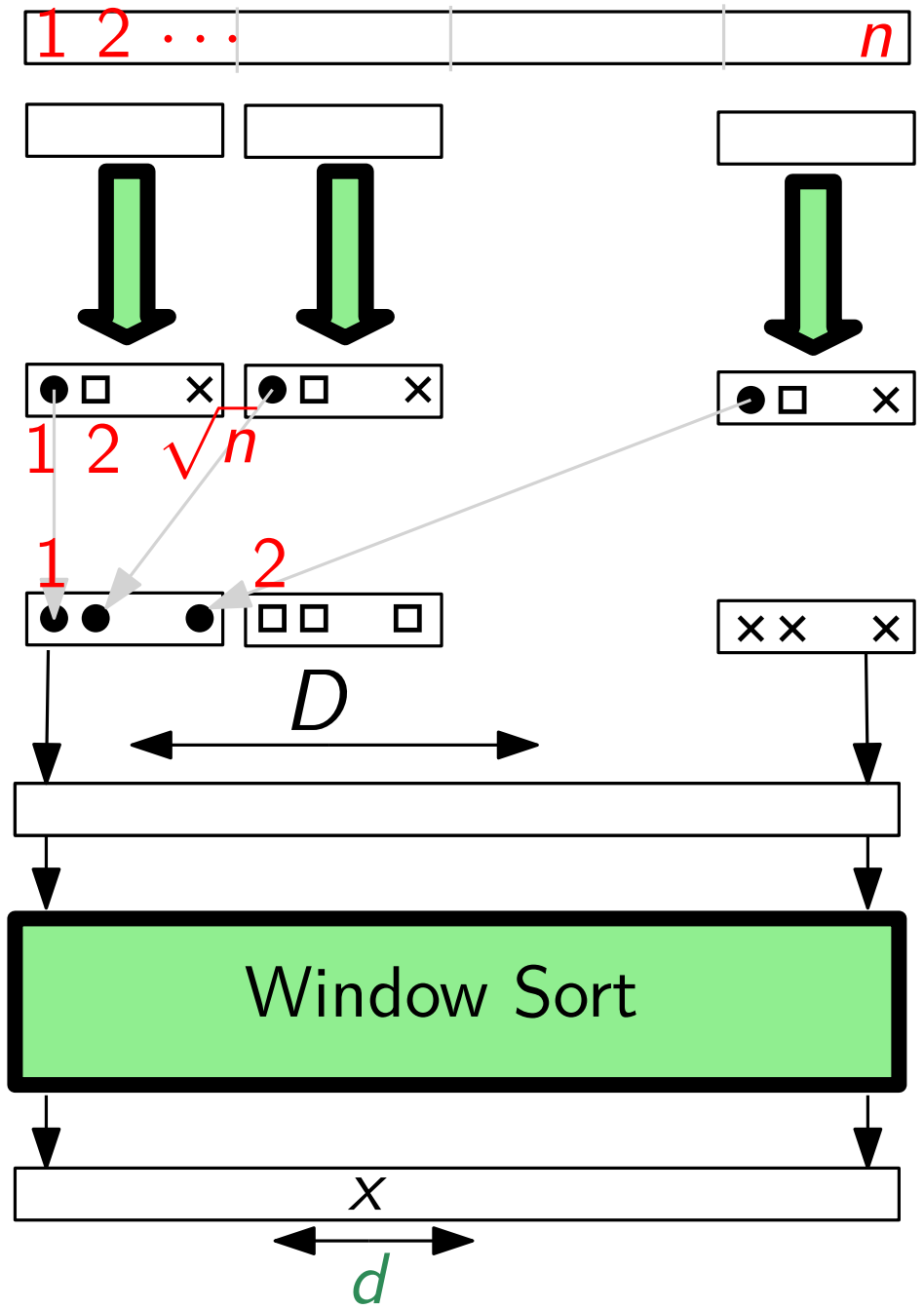
$O(n^{3/2})$

Simple Faster Algo



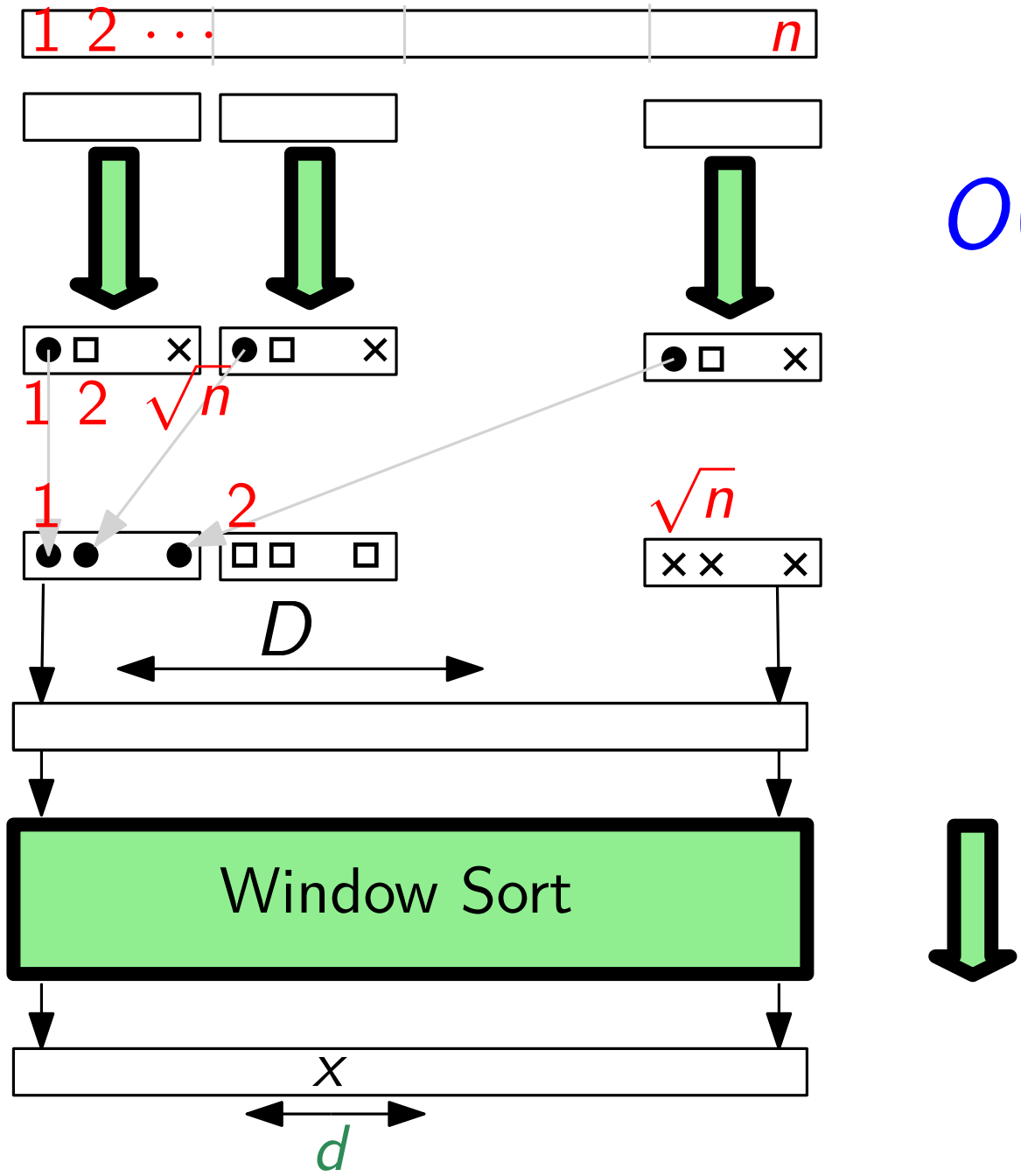
$O(n^{3/2})$

Simple Faster Algo



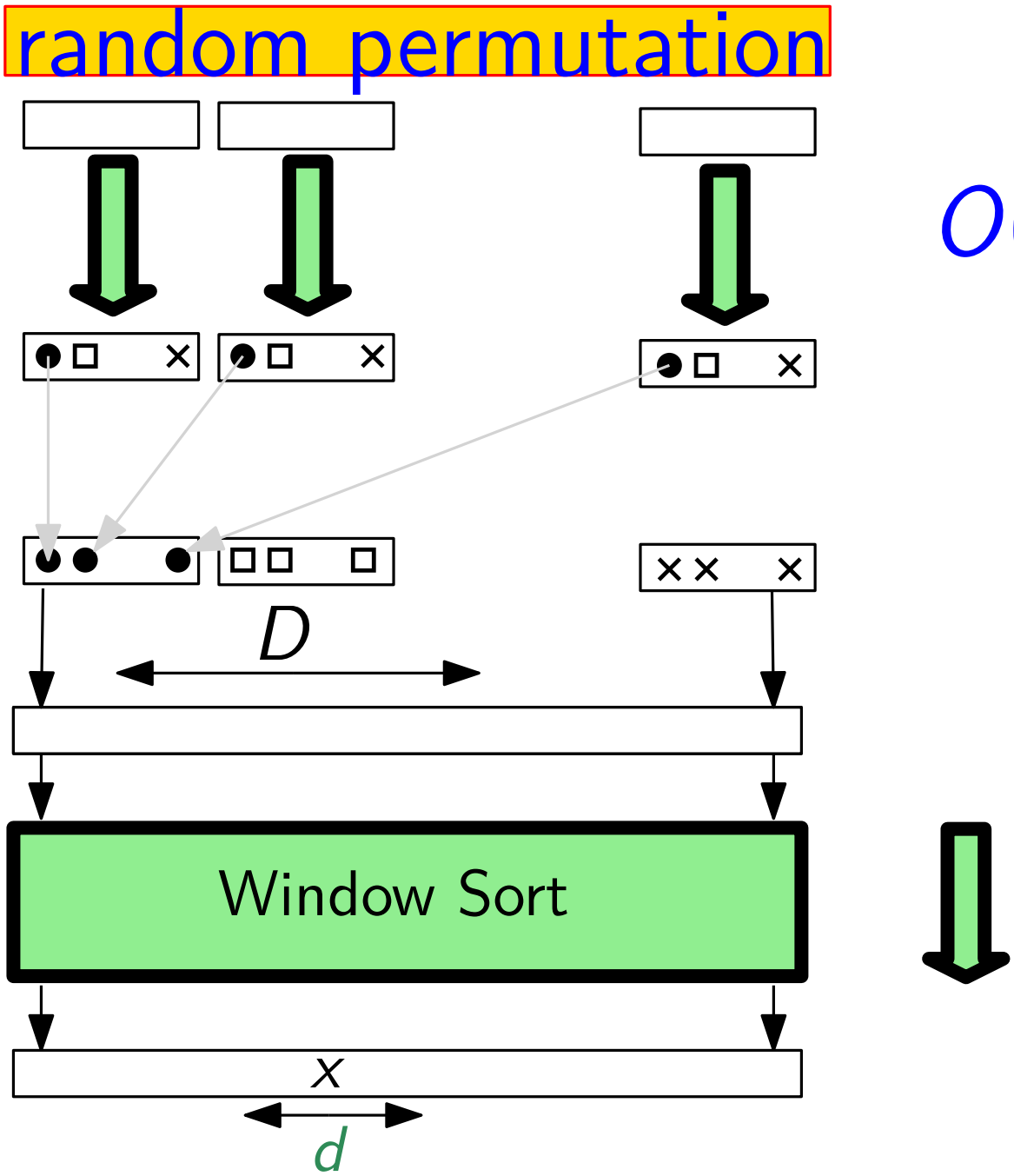
$O(n^{3/2})$

Simple Faster Algo



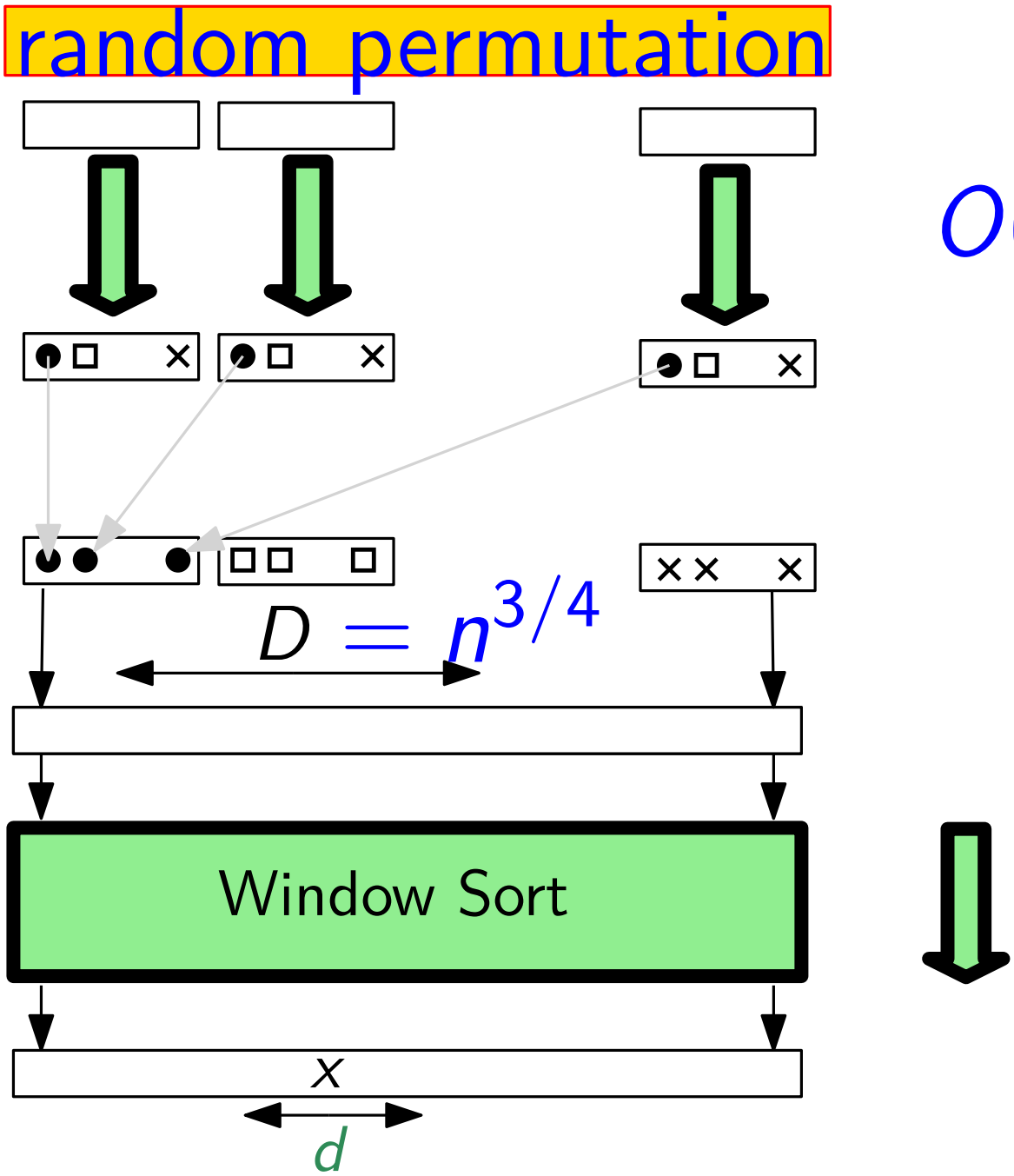
$O(n^{3/2})$

Simple Faster Algo



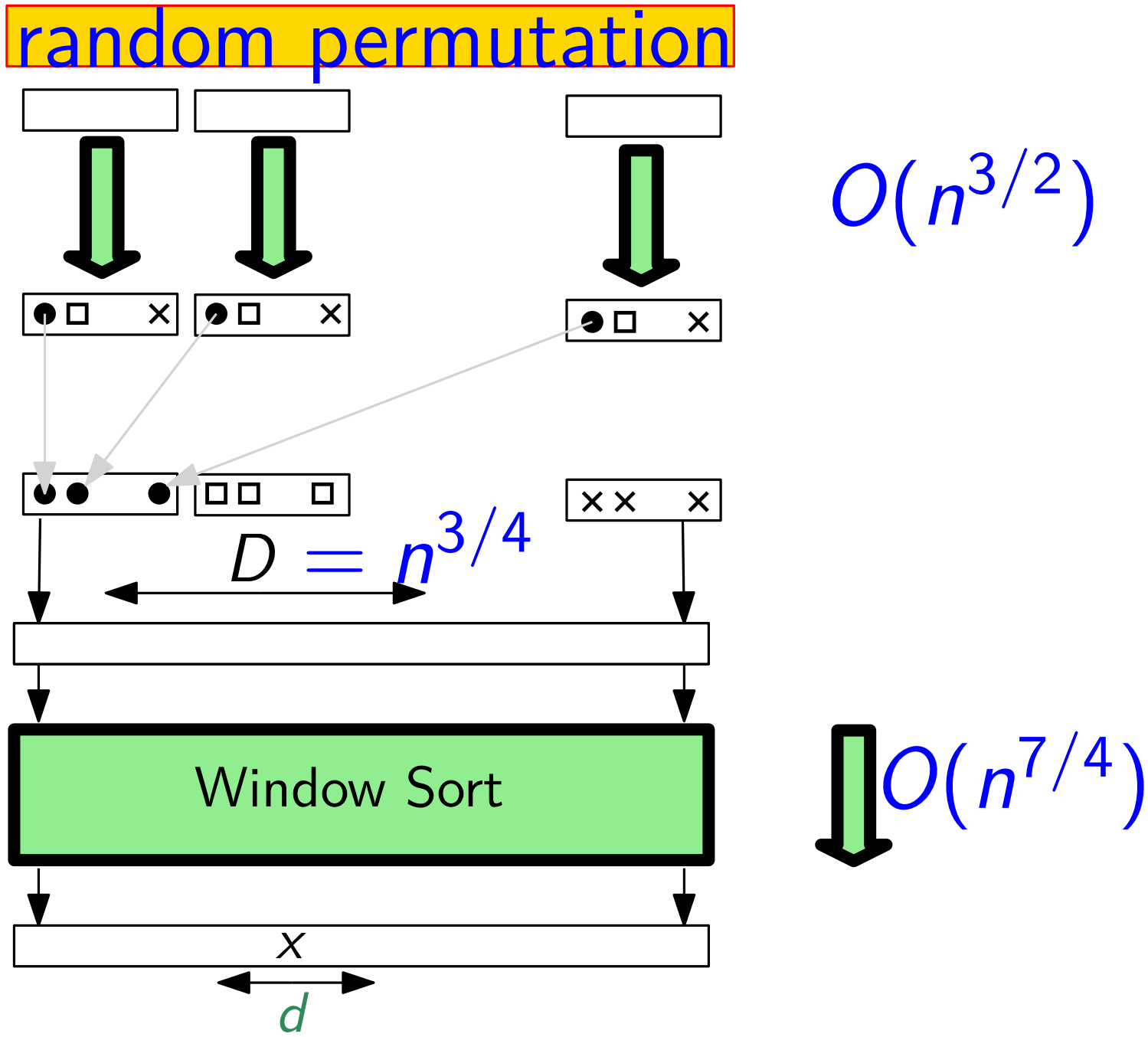
$O(n^{3/2})$

Simple Faster Algo

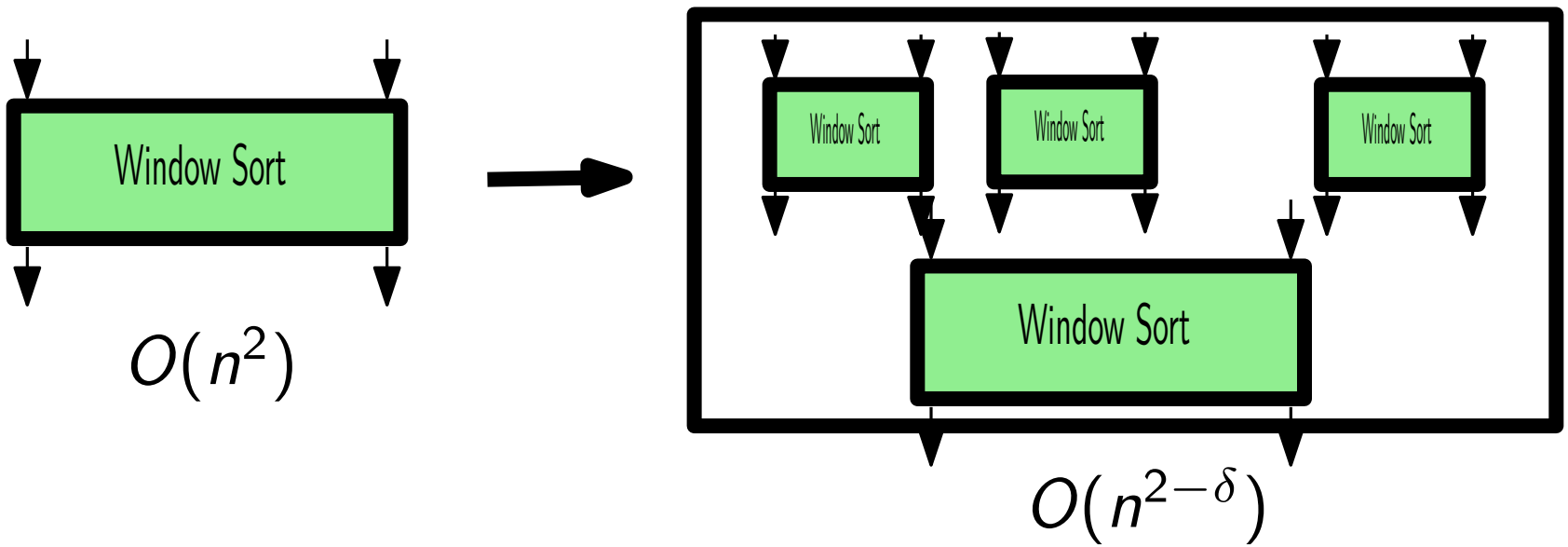


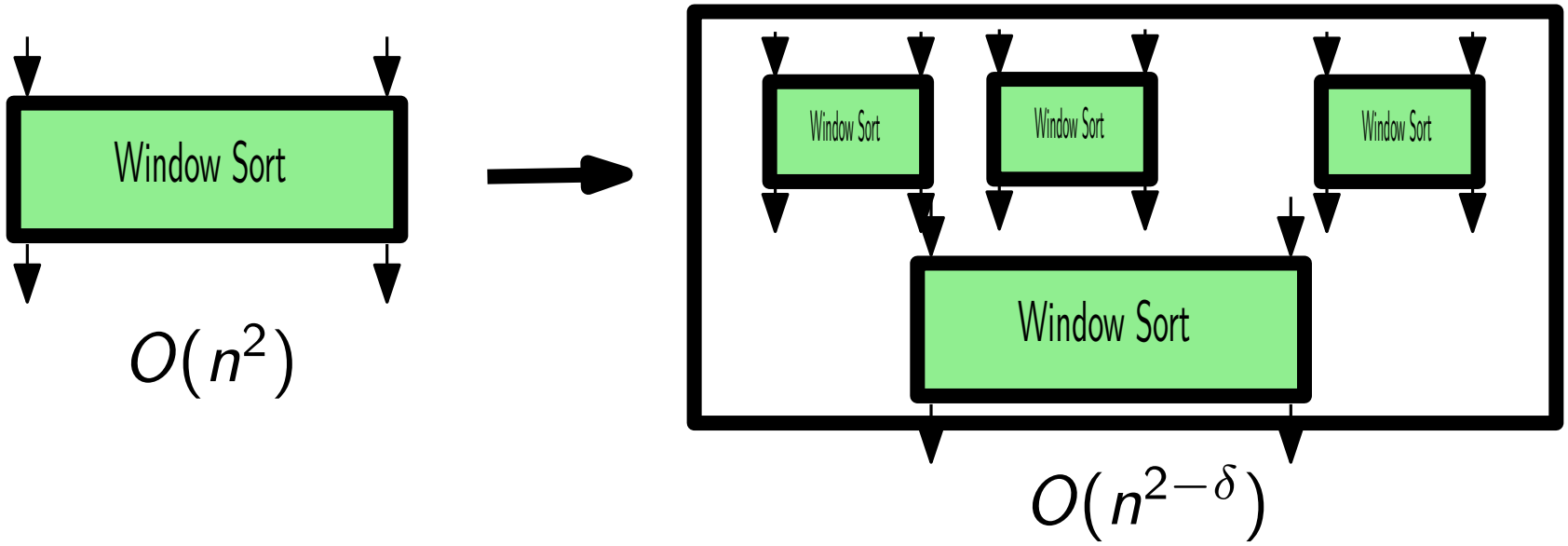
$O(n^{3/2})$

Simple Faster Algo



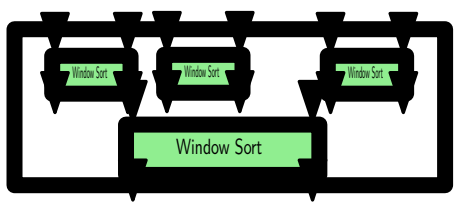
That was the **simple** version...


Window Sort
$O(n^2)$
Window Sort
Window Sort
Window Sort
Window Sort
$O(n^{2-\delta})$

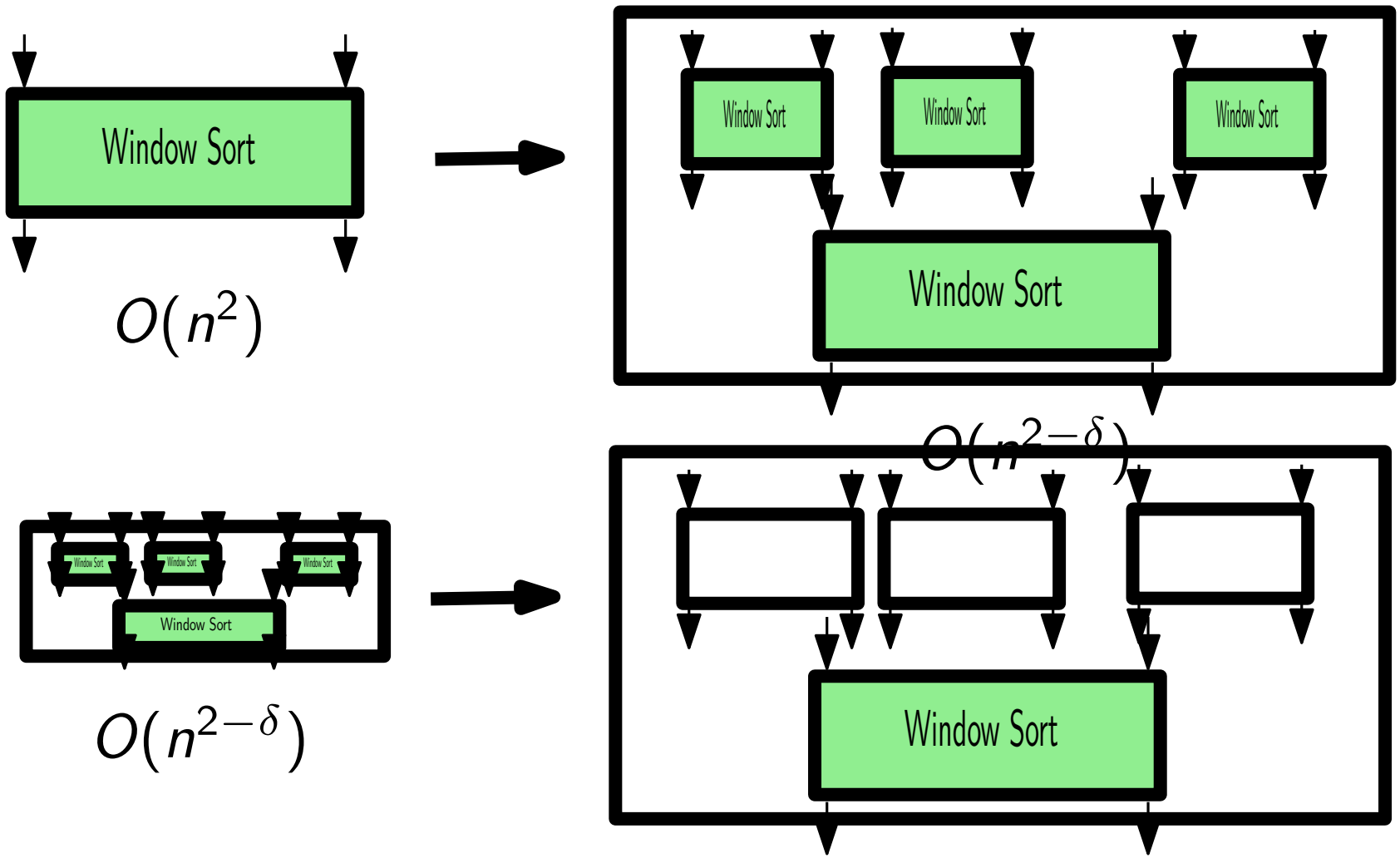


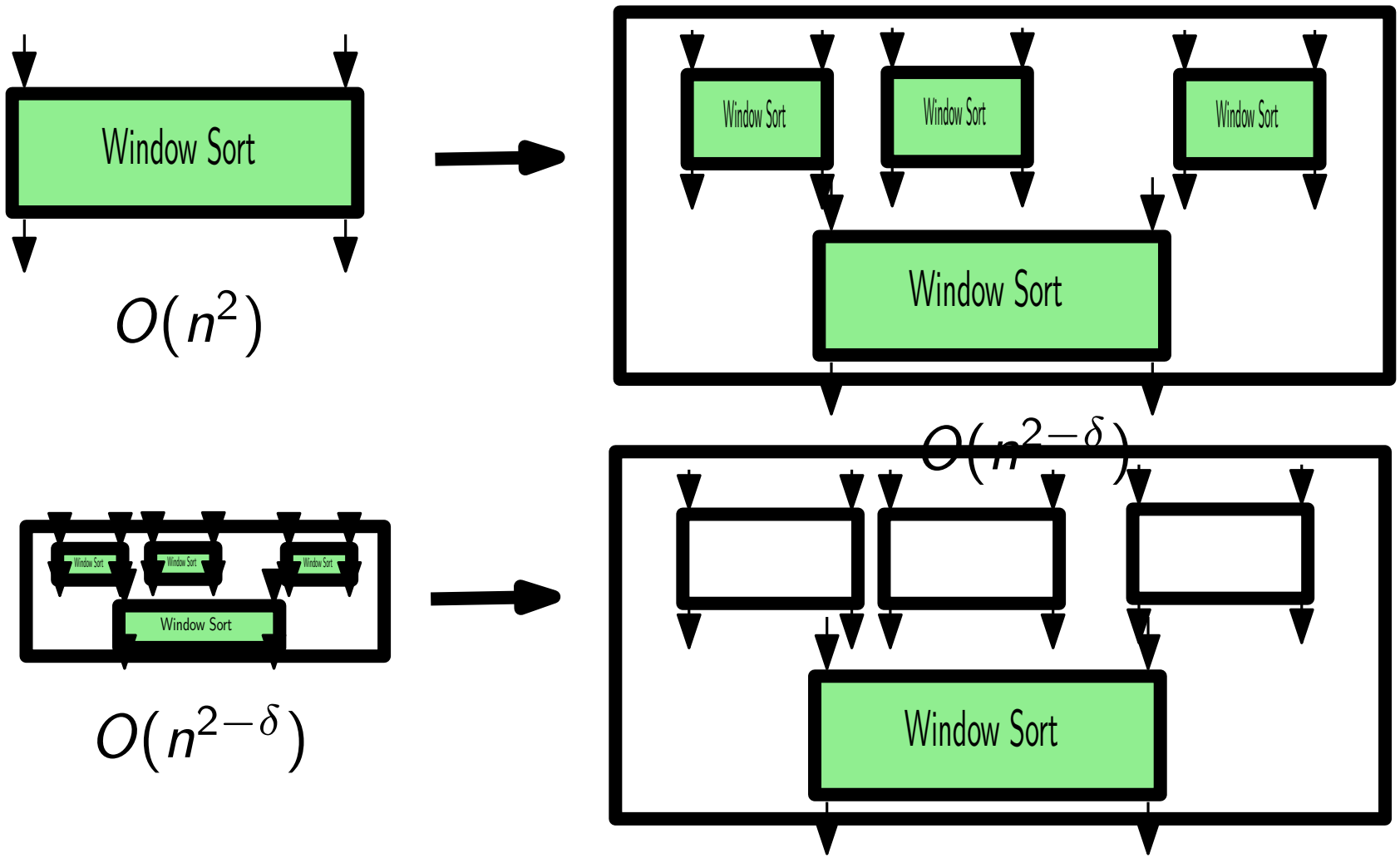
$O(n^2)$

$O(n^{2-\delta})$



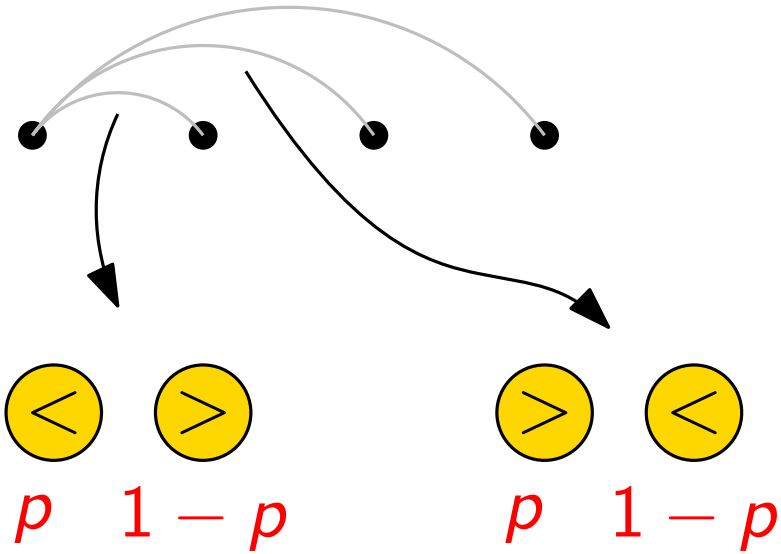
$O(n^{2-\delta})$

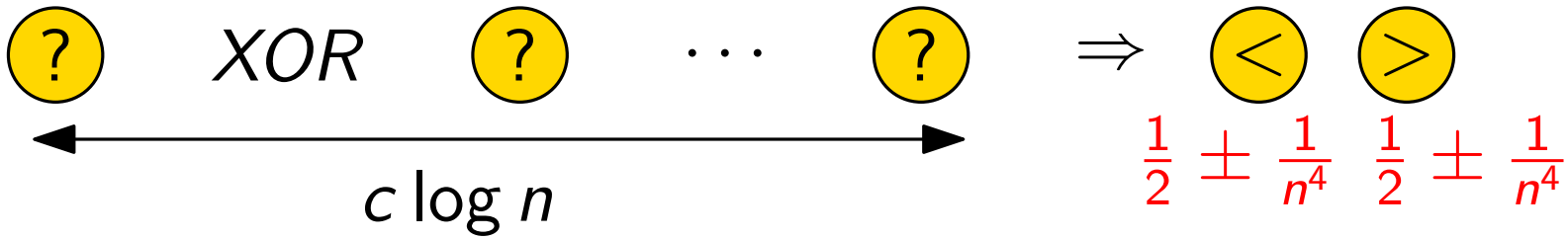

Window Sort
$O(n^2)$
Window Sort
Window Sort
Window Sort
Window Sort
$O(n^{2-\delta})$
Window Sort
Window Sort
Window Sort
Window Sort
$O(n^{2-\delta})$
Window Sort


Window Sort
$O(n^2)$
Window Sort
Window Sort
Window Sort
Window Sort
$O(n^{2-\delta})$
Window Sort
Window Sort
Window Sort
Window Sort
$O(n^{2-\delta})$
Window Sort
$\cdots O(n^{3/2})$

Part II: Derandomization

Comparisons $\Rightarrow$ Randomness

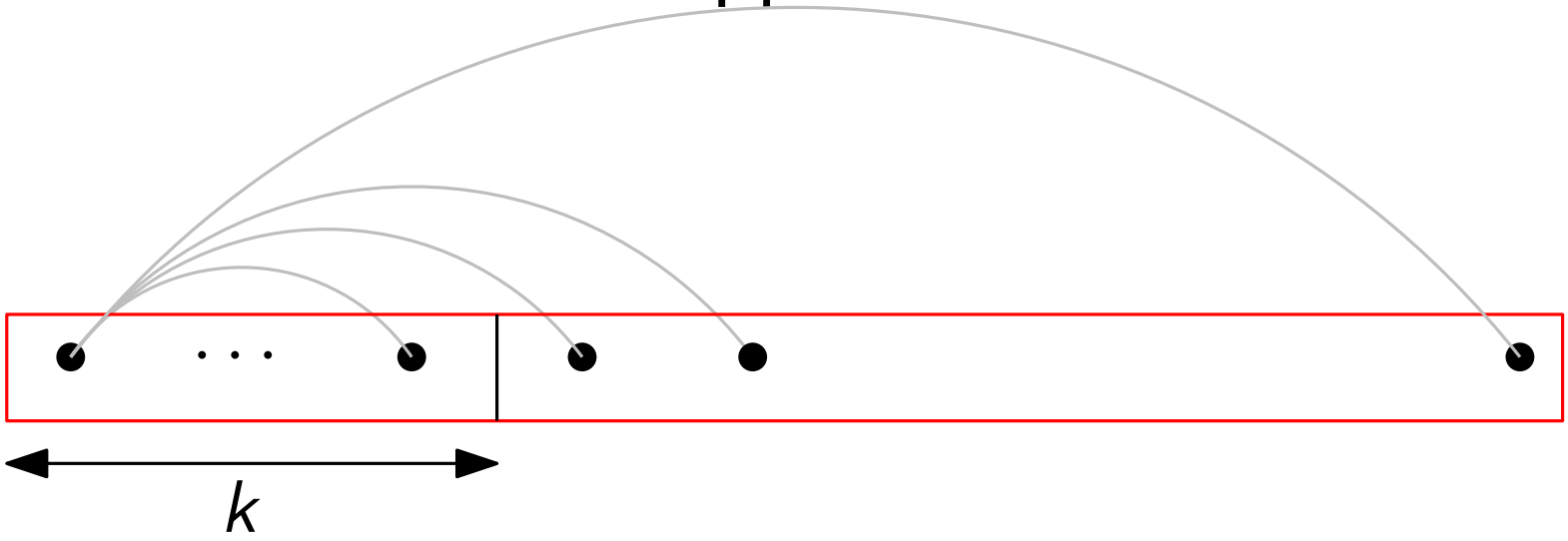


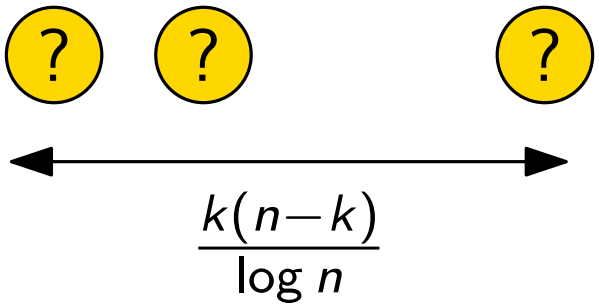


Comparisons

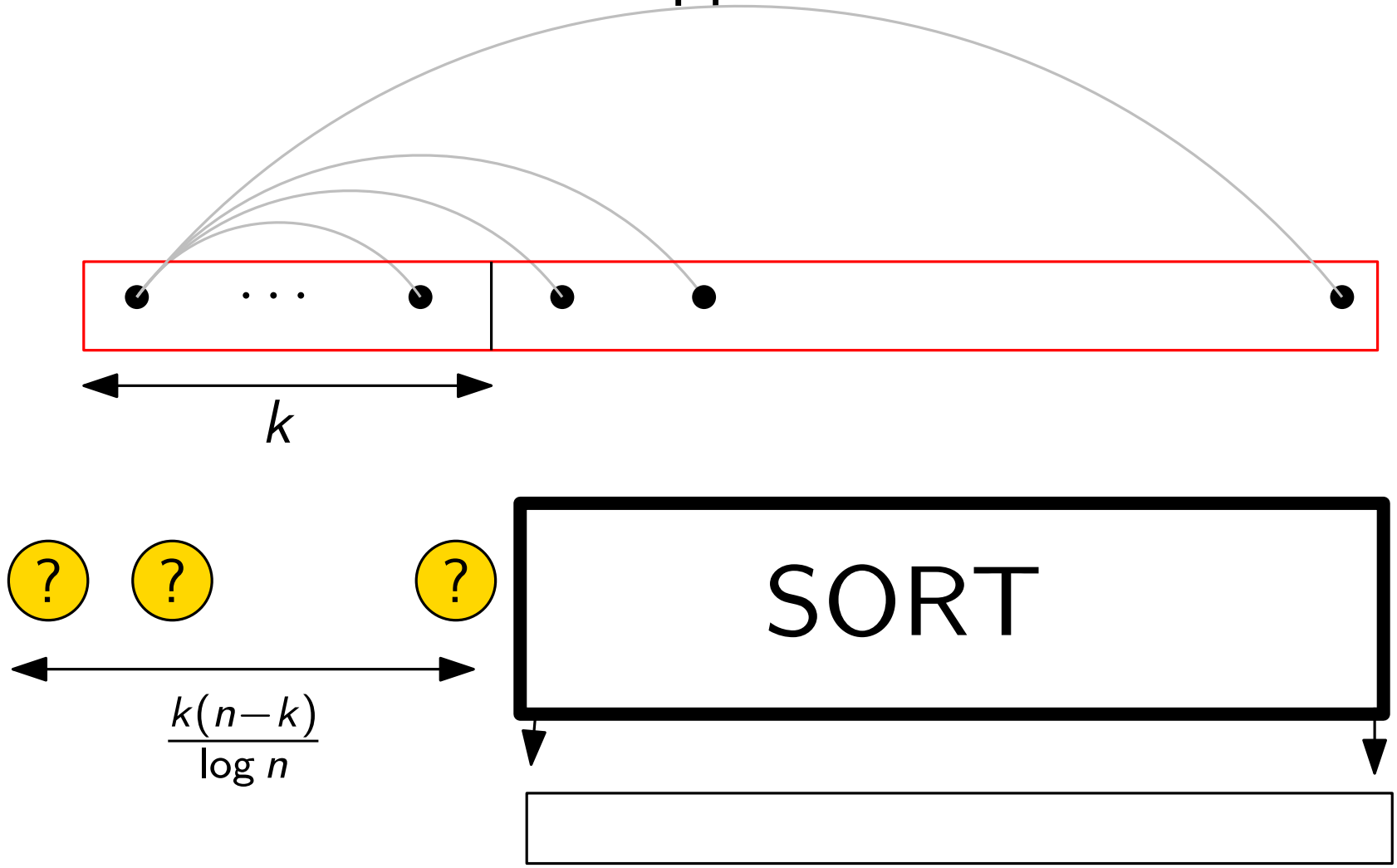
One random bit

Naive Approach

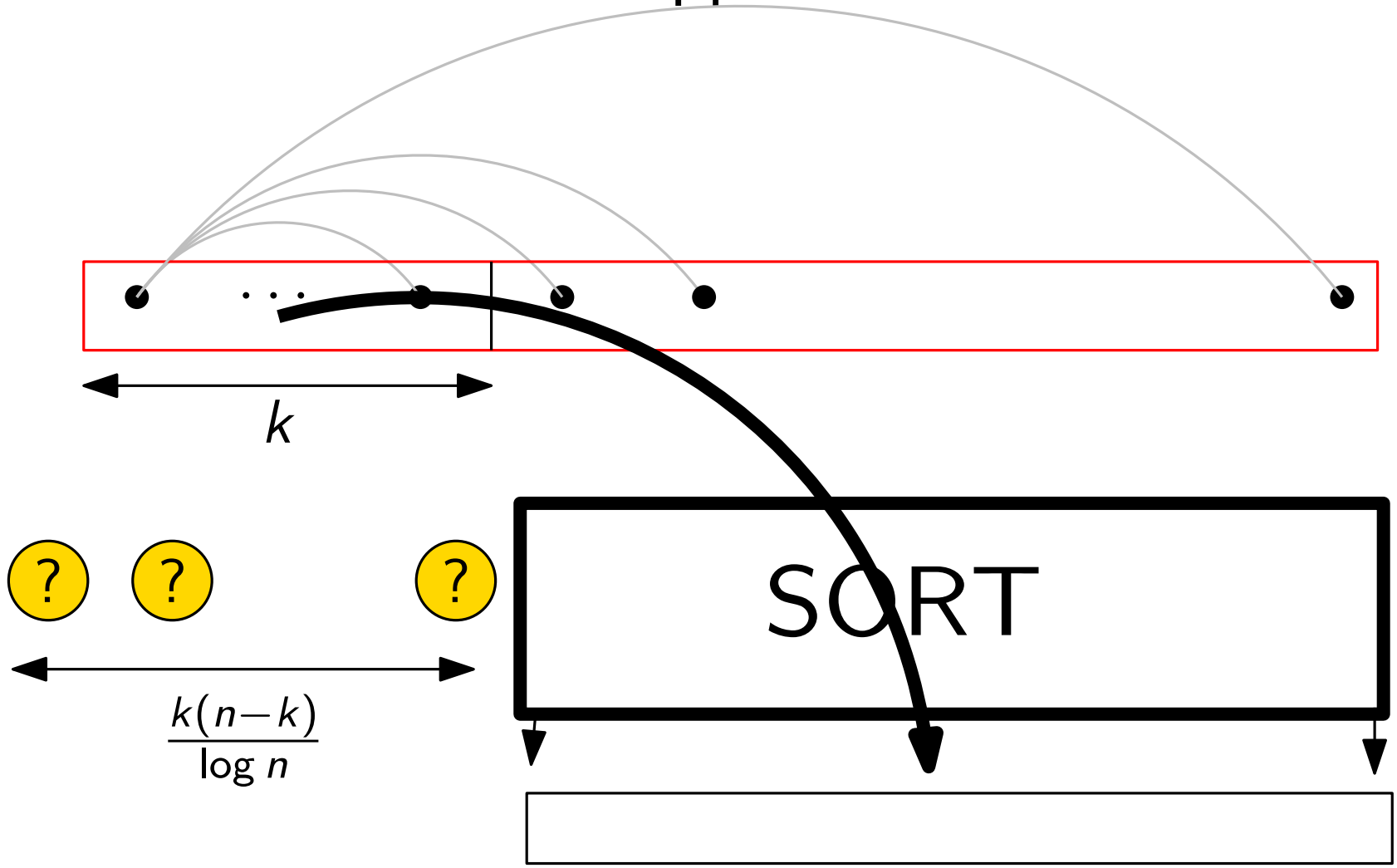




Naive Approach



Naive Approach



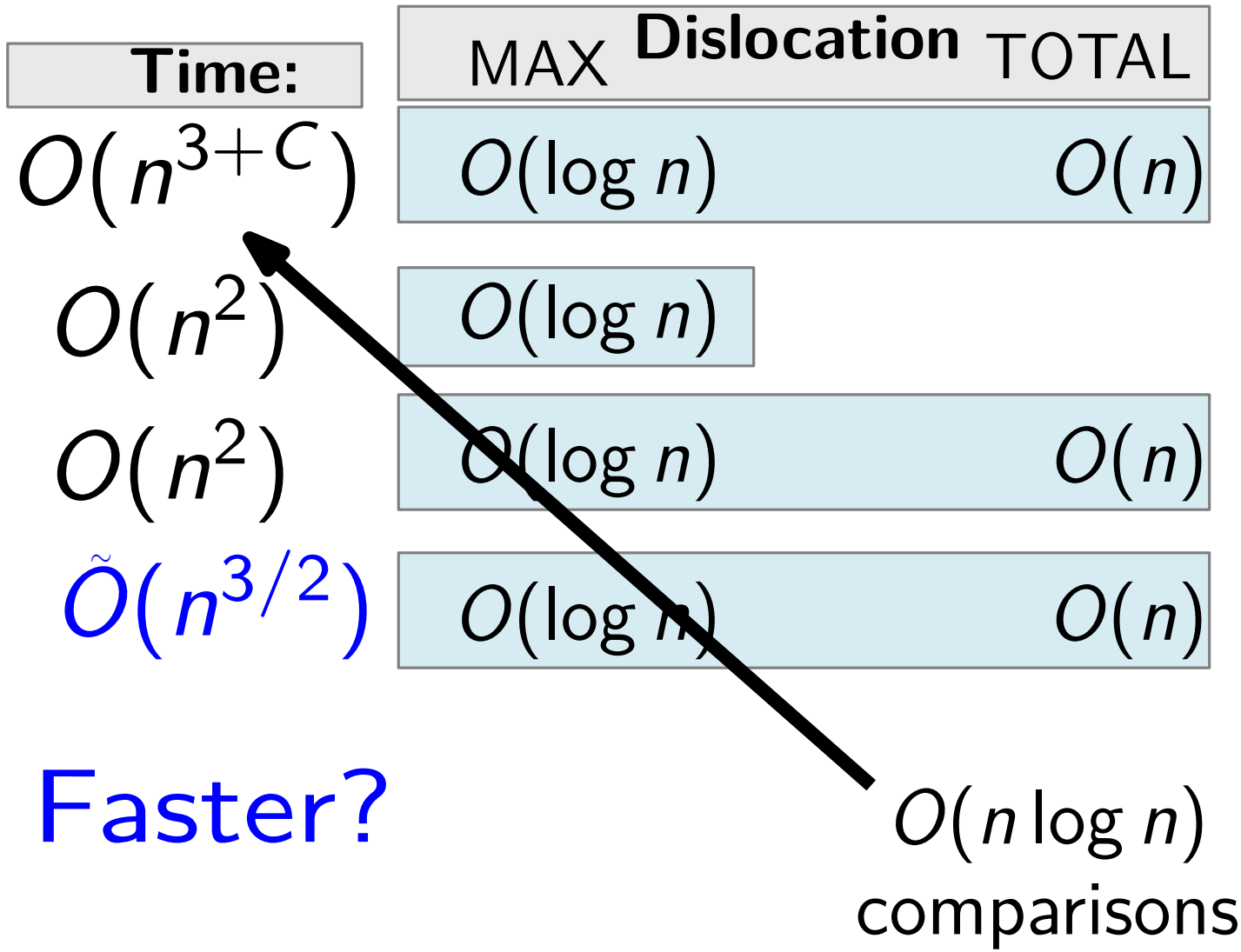
REINSERT

Open Questions

Time:	Dislocation MAX	Dislocation TOTAL
$O(n^{3+C})$	$O(\log n)$	$O(n)$
$O(n^2)$	$O(\log n)$	
$O(n^2)$	$O(\log n)$	$O(n)$
$O(n^{3/2})$	$O(\log n)$	$O(n)$

Braverman & Mossel (SODA'08)
Klein, Penninger, Sohler, Woodruff (ESA'11)
Geissmann, Leucci, Liu, Penna (ISAAC'17)

Open Questions



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Open Questions

Time:	Dislocation MAX	TOTAL	
$O(n^{3+C})$	$O(\log n)$	$O(n)$	$p < 1/16$
$O(n^2)$	$O(\log n)$		
$O(n^2)$	$O(\log n)$	$O(n)$	
$O(n^{3/2})$	$O(\log n)$	$O(n)$	

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Open Questions

Time:	Dislocation MAX	TOTAL	
$O(n^{3+C})$	$O(\log n)$	$O(n)$	$p < 1/16$
$O(n^2)$	$O(\log n)$		
$O(n^2)$	$O(\log n)$	$O(n)$	
$O(n^{3/2})$	$O(\log n)$	$O(n)$	

Any $p < 1/2$?

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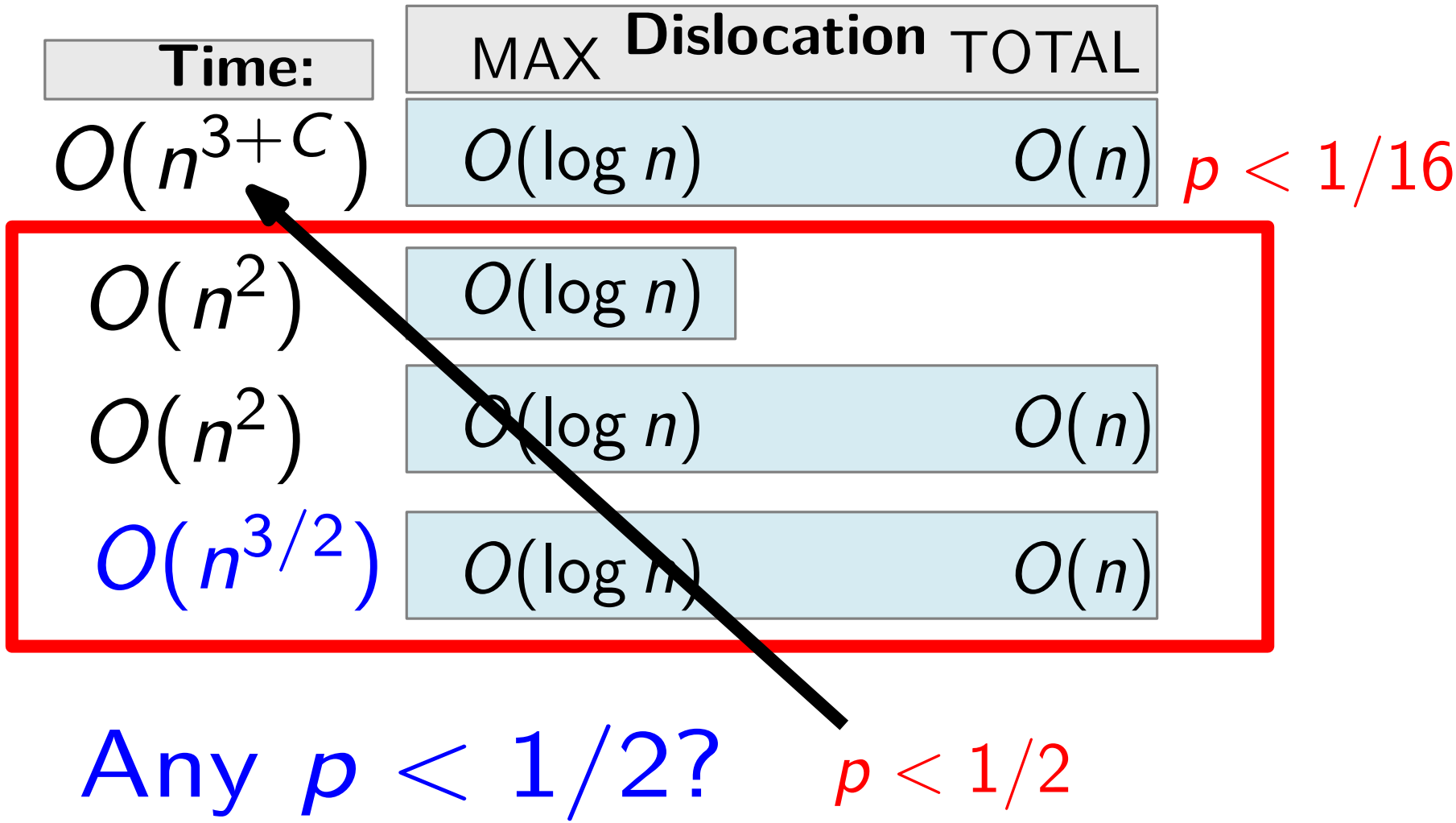
Open Questions



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Open Questions

Time:	Dislocation MAX	TOTAL
$O(n^{3+C})$	$O(\log n)$	$O(n)$
$O(n^2)$	$O(\log n)$	
$O(n^2)$	$O(\log n)$	$O(n)$
$O(n^{3/2})$	$O(\log n)$	$O(n)$

Other error models?

Braverman & Mossel (SODA'08)
Klein, Penninger, Sohler, Woodruff (ESA'11)
Geissmann, Leucci, Liu, Penna (ISAAC'17)

Tahnk You